UNIT-5

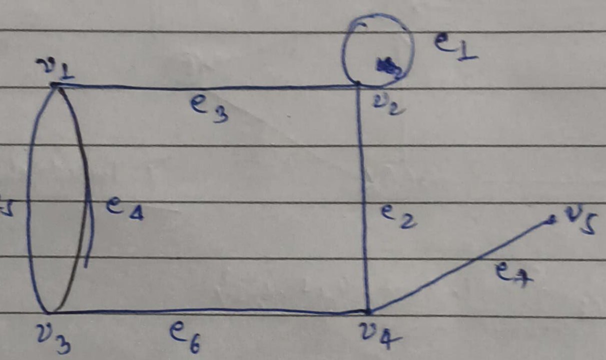
Graph

A Graph can be defined as group of vertices and edge that are used to connect ~~these~~ vertice a graph consist of two things $G = (V, E)$

$V = \{v_1, v_2, v_3, v_4, v_5\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$

(i) A set V of element called Node (or point or vertex)

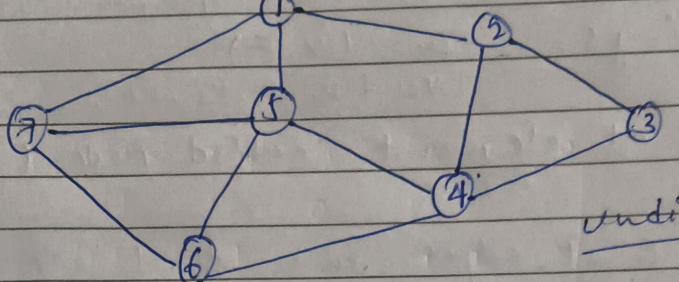
(ii) A set E of edges such that each edge in E is unique pair of vertex (u, v)



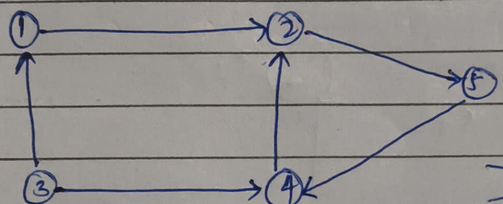
- ↳ Adjacent Node
- ↳ Degree of node
- ↳ self loop
- ↳ cycle
- ↳ parallel ~~cycle~~ edge
- ↳ Isolate node
- ↳ path

Graph can be two type

1. Undirected Graph
2. Directed Graph



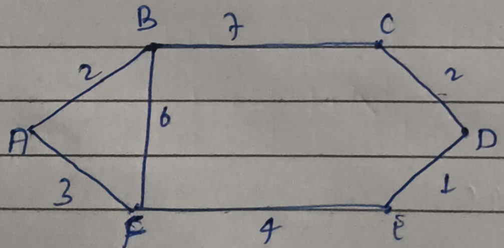
Undirected



Directed

V(G) = { 1, 2, 3, 4, 5, 6, 7 }

V(E) = { (1, 2), (1, 5), (7, 1), (7, 6), (5, 6), (6, 4), (3, 2), (3, 4) }

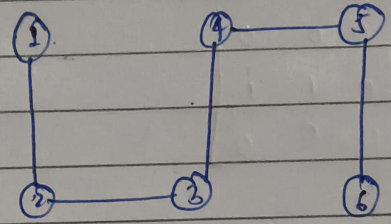


weighted Graph

Representation of Graph

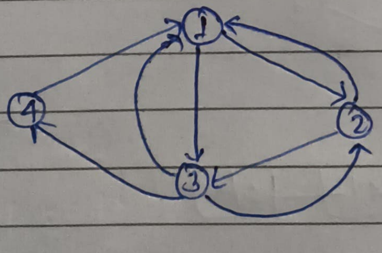
Usually a graph can be represented in many way some of these representation are

1. set Representation
2. Sequential Representation
   (i) Adjacency Adjancey
   (ii) Incidence
3. Link Representation



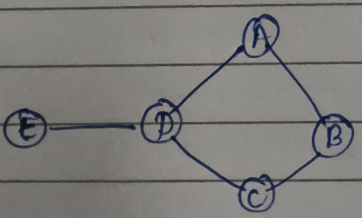
$V(G) = \{1, 2, 3, 4, 5, 6\}$

$E(G) = \{(1,2), (2,3), (3,4), (4,5), (5,6)\}$

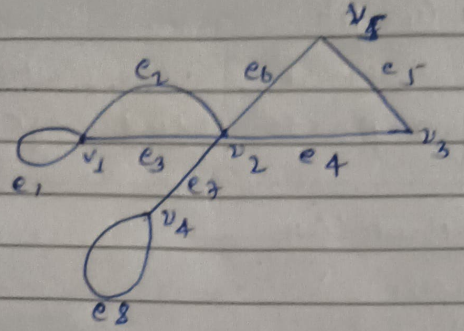


Adjancecy ~~matrix~~

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 1 & 0 & 0 & 0 \end{array}$$



$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 1 & 0 & 1 & 0 \\ B & 1 & 0 & 1 & 0 & 0 \\ C & 0 & 1 & 0 & 1 & 0 \\ D & 1 & 0 & 1 & 0 & 1 \\ E & 0 & 0 & 0 & 1 & 0 \end{array}$$

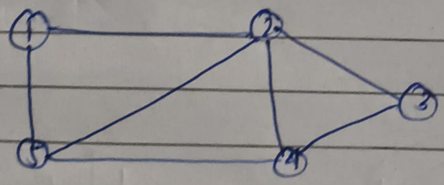


Incident Matrix =

$$\begin{array}{c|cccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \\ \hline v_1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ v_3 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ v_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ v_5 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{array}$$

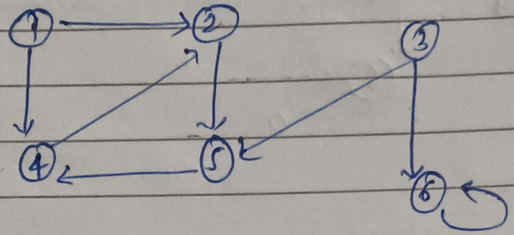
Linked Representation

Undirected Graph



[1|] → [2|] → [5|\]
[2|] → [1|] → [3|] → [4|] → [5|\]
[3|] → [2|] → [4|\]
[4|] → [2|] → [3|] → [5|\]
[5|] → [1|] → [2|] → [4|\]

Linked Representation of Directed Graph



1 → 2 → 4 | \

2 → 5 | \

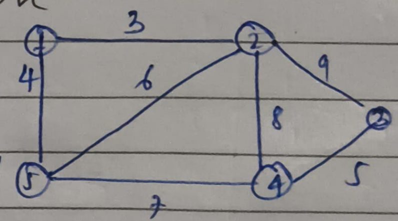
3 → 5 → 6 | \

4 → 2 | \

5 → 4 | \

6 → 6 | \

weighted indirected Graph



1 → 3 | 2 → 4 | 5 | \

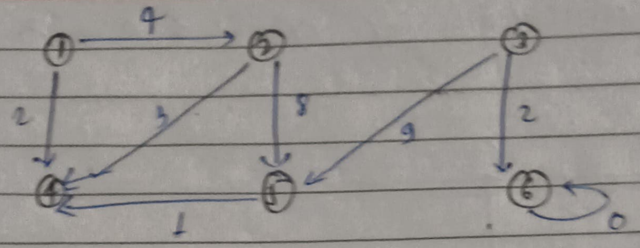
2 → 3 | 1 → 9 | 3 → 8 | 4 → 6 | 5 | \

3 → 9 | 2 → 5 | 4 | \

4 → 8 | 2 → 5 | 3 → 7 | 5 | \

5 → 4 | 1 → 6 | 2 → 7 | 4 | \

Weighted Directed Graph



| 1 | | — | 4 | 2 | | — | 2 | 4 | \ |

| 2 | | — | 8 | 5 | \ |

| 3 | | — | 9 | 5 | | — | 2 | 6 | \ |

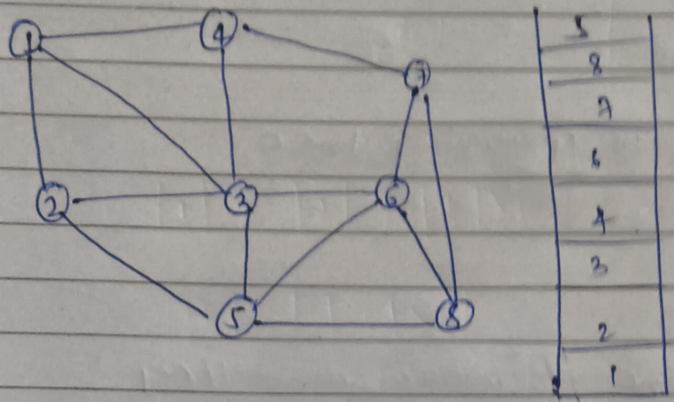
| 4 | | — | 3 | 2 | \ |

| 5 | | — | 1 | 4 | \ |

| 6 | | — | 0 | 6 | \ |

Traversing of Graph

(1) Depth-First search (DFS) LIFO → (stack)



order

1, 4, 7, 8, 5, 6, 3, 2

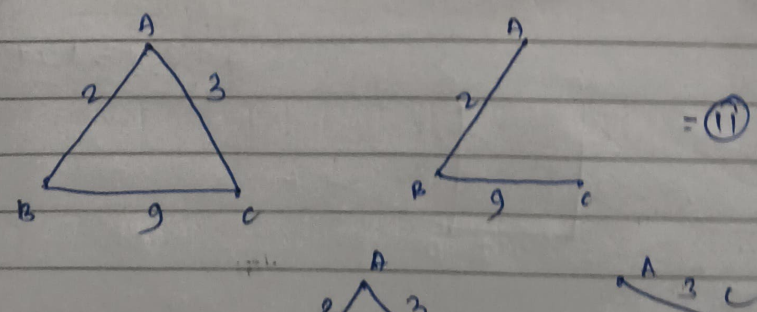
(2) Breadth-First search (BFS) → (Queue)

~~1~~	~~2~~	3	~~4~~	~~5~~	6	~~7~~	~~8~~

1, 2, 3, 4, 5, 6, 7, 8

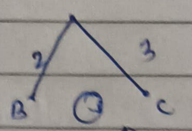
Spanning Tree

A spanning tree is a subset of graph which has all vertex covered with minimum number of edge



Minimum Spanning Tree

In a weighted graph graph a minimum spanning tree that has a minimum weight than other spanning tree of graph



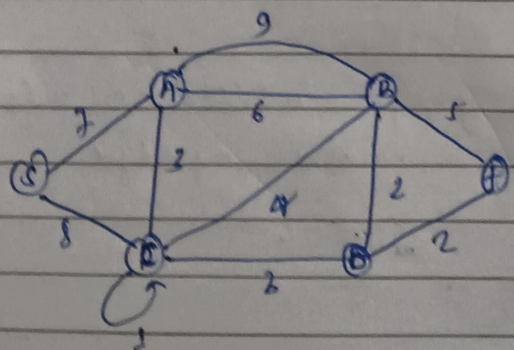
Minimum spanning tree

Properties

i. Spanning Tree (n-1) edge where n = vertices
ii. Number of vertice and edge in all spanning tree
iii. There are nocycle in spanning trees
iv. If Graph is complete then n^{n-2} spanning trees

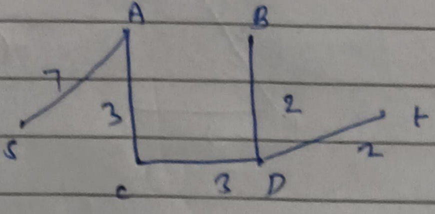
Kruskal's Algorithm:→

① Remove all parallel and loop edge
② sort the edges in non decreasing order

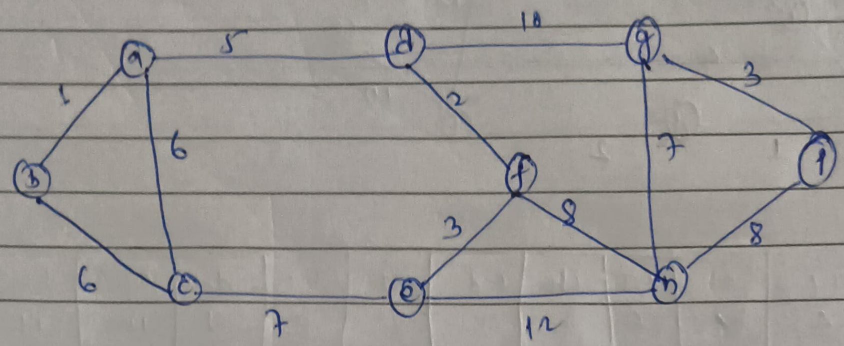


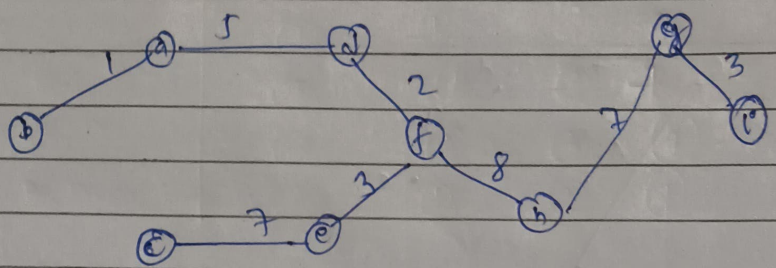
BD	DF	AC	CD	CB	BF	AB	AS	CS
2	2	3	3	4	5	6	7	8

7+3+3+2+2 = 17

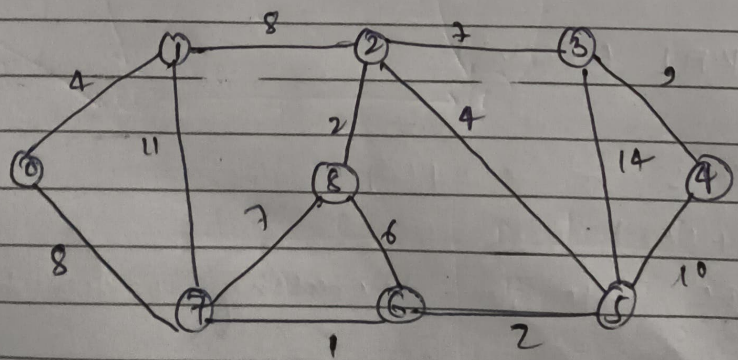


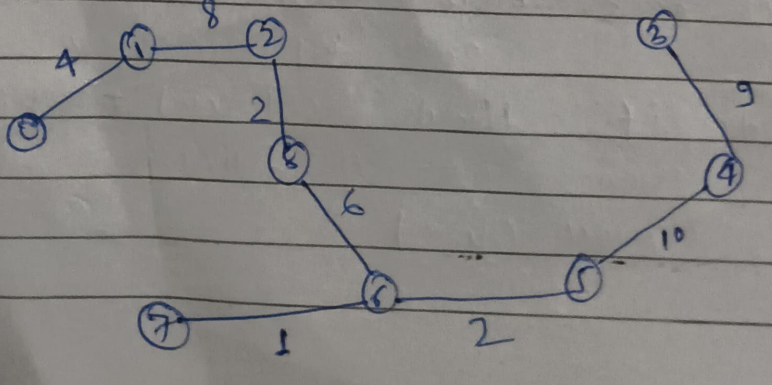
Prim's Algorithm

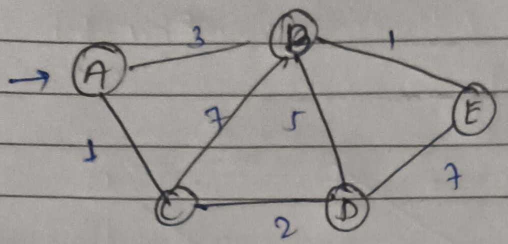


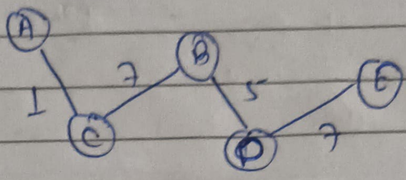


1 + 5 + 2 + 8 + 7 + 3 + 7 + 3 ⇒ 36 ✓

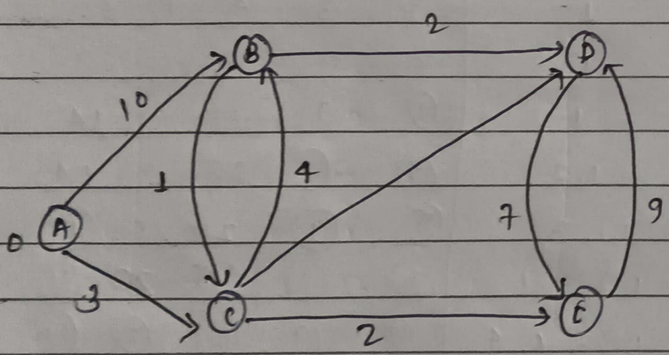




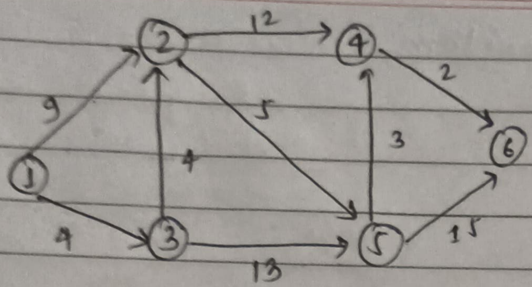




Source	Destination			
A	B	C	D	E
A	∞	∞	∞	∞
A	3	1	∞	∞
A, C	3	1	3	∞
A, C, B	3	1	3	4
A, C, B, D	3	1	3	4
A, C, B, D, E	3	1	3	4



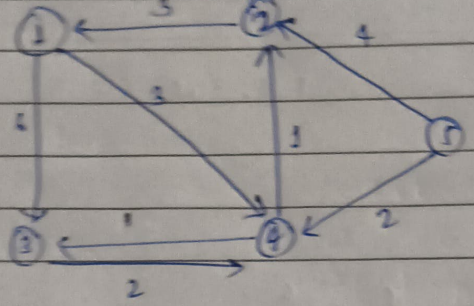
Source	Destination			
A	B	C	D	E
A	∞	∞	∞	∞
A	10	3	∞	∞
A, C	7	3	11	5
A, C, E	7	3	11	5
A, C, E, B	7	3	9	5
A, C, E, B, D				



1,3,2,5,4,6

Source	Destination				
1	2	3	4	5	6
1	∞	∞	∞	∞	∞
1	9	4	∞	∞	∞
1, 3	9	4	∞	17	∞
1, 3, 2, 5	9	4	16	17	∞
1, 3, 2, 5, 4,	9	4	16	17	18
1, 3, 2, 5, 4, 6					

floyd- warshall's Algorithm



$$D_0 = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$D^1 = \begin{array}{c|c} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$D^2 = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$$D^3 = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$$D4 = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ 7 & 3 & 0 & 2 & \infty \\ 4 & 1 & 2 & 0 & \infty \\ 6 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$

$$D5 = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ 6 & 3 & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 6 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$

Weighted Graph

A graph is said to be a weighted graph if all the phase in it are labelled with some number

Self loop:-

If there is an edge whose starting and end vertex are same that is (v_1, v_1) is an edge then is called self loop or simply loop.

Parallel Edges:-

If there are more than one edges between the same pair of vertex then they are known as parallel edge.

Adjacent Vertex:

A vertex u is adjacent to other vertex v if there is an edge from u to v.

Incidence

In an undirected graph the edge (u, v) is incident on vertex u and v in a direct graph the edge (u, v) is incident from ~~ff~~ node u and is incident to node v

Degree of Vertex

The degree of vertex is the number of edge incident to that vertex in an undirect graph the number of edge connected to a node is called the degree of that node.

Weighted Graph

A graph is said to be a weighted graph if all the phase in it are leveled with same number

Self loop :-

If there is an edge whose starting & end vertex are same that is (v_2, v_2) is an edge then is called self loop or simply loop.

Parallel Edges :-

If there are more than one edges between the same pair of vertex then they are known as parallel edge.

Adjacent Vertex :

A vertex u is adjacent to other vertex v if there is an edge from u to v.

Incidence

In an undirected graph the edge (u, v) is incident on vertex u and v in a direct graph the edge (u, v) is incident from ~~fr~~ node u and is incident to node v

Degree of Vertex

The degree of vertex is the number of edge incident to that vertex in an undirect graph the number of edge connected to a node is called the degree of that node.

↳ In degree : In degree of a vertex is the number of edge comming to that vertex or in other word edge is incident to it.

↳ out degree → The outdgree of a vertex is the number of egde going outside from that vertex or in other word the edge incident from it.

Simple Graph

A graph of directed graph which does not have any self loop or parallel edge is called simple graph.

multigraph

A graph which has either a self loop or parallel edge or both is called a multigraph

Complete Graph

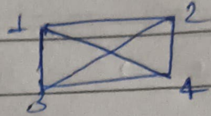
A graph is complete graph if each vertex is a adjacent to every other vertex in graph or we can say that there is an edge between any pair of nodes in the graph

Planer Graph

A graph is planer if it can with draw in a plaine without any two edge intersecting

Regular Graph:

A graph is regular if every node is a adjacent to the same numbers of nodes

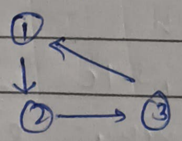


Connected Graph

In a graph G to vertex v_1 and v_2 are said to be connected if there is path in G from v_1 to v_2 or v_2 to v_1. A graph is said be connected if there is a path from any node of graph to any other node i.e; for every pair of distinict vertex in G there is a path.

Strongly Connected Graph

A directed graph is said to be strongly connected graph if for every pair of the distincts vertex in G there is a path



Cycle

If there is a path containing one or more edge which starts from a vertex and terminate into the same vertex then the path is called cycle

A cycle graph

If a graph does not have any cycle then it is called acycle graph