

# Unit - 02

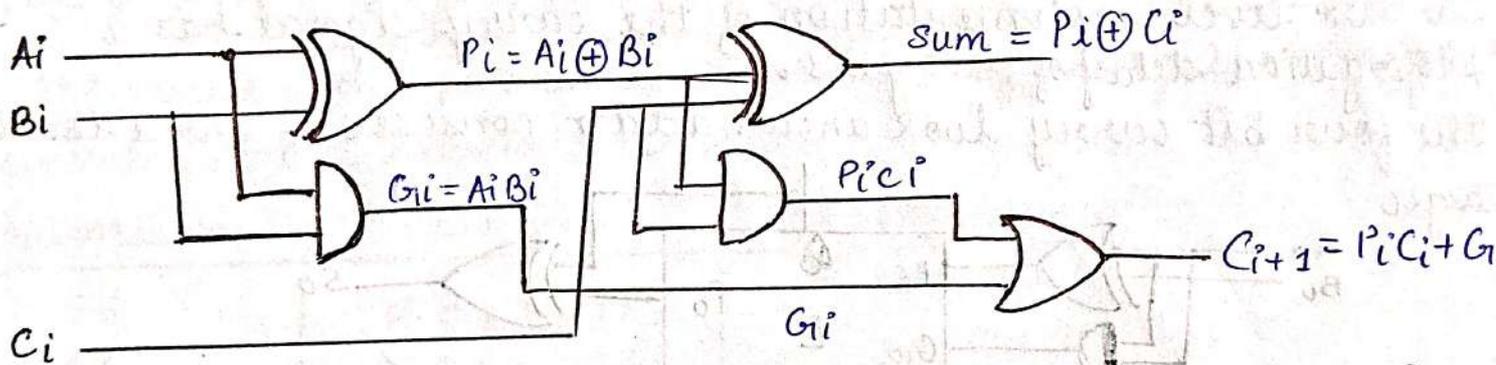
## Arithmetic & Logic Unit

### Look ahead Carry Adder

To reduce the propagation delay

- To overcome the problem of excessive delay in ripple carry Adder, a fast adder known as look ahead carry adder can be designed. In this adder the carry-in for various stages can be generated directly by the logical expressions.
- The general method for design the fast adder is to reduce the time required to form carry signals.
  - It uses the logical gates to look at lower order bits of the data to see if a higher order carry is to be generated.
  - It uses two functions
    - Carry generate
    - Carry propagate

Following figure shows the full adder circuit used to add a operand bits



From the given circuit  $Sum = P_i \oplus C_i$  and Carry  $C_{i+1} = P_i C_i + G_i$

- $G_i$  is known as carry generated signal
- A carry  $C_{i+1}$  is generated whenever  $G_i = 1$ , regardless of the input carry.
- $P_i$  is known as carry propagate whenever  $P_i = 1$  the input carry is propagated to the output carry
- The boolean expression for the carry output of various stages can be written as follows :-

$i=0$

$$C_1 = G_0 + P_0 C_0$$

$$C_1 = [A_0 B_0] + [A_0 \oplus B_0] C_0$$

$$C_{i+1} = P_i C_i + G_i$$

$i=0$

$$C_1 = P_0 C_0 + G_0$$

$i=1$

$$C_2 = P_1 C_1 + G_1$$

$$= P_1 [P_0 C_0 + G_0] + G_1$$

$$C_2 = P_1 P_0 C_0 + P_1 G_0 + G_1$$

$i=2$

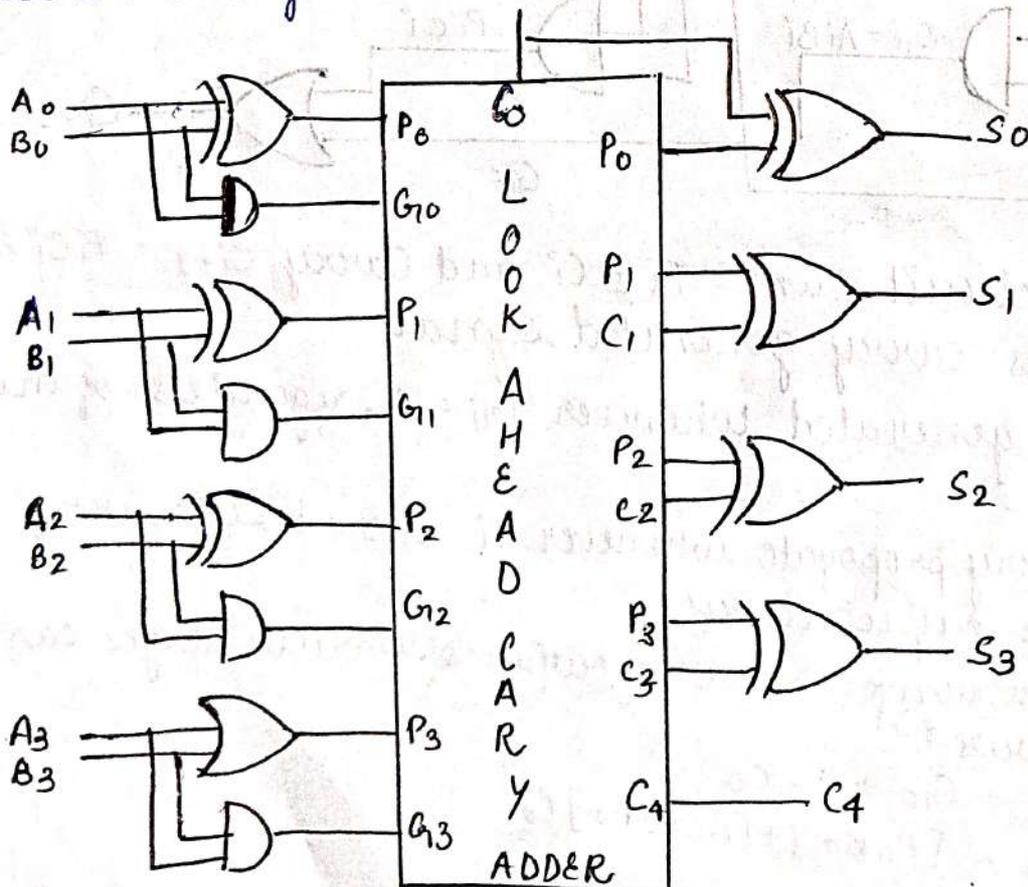
$$C_3 = P_2 C_2 + G_2$$

$$= P_2 [P_1 P_0 C_0 + P_1 G_0 + G_1] + G_2$$

$$C_3 = P_2 P_1 P_0 C_0 + P_1 P_2 G_0 + G_1 P_2 + G_2$$

From the above expression each carry signal is expressed as a direct SOP form  $C_0$ .

- The two level implementation of the carry signal has a propagation delay of two gates.
- The four bit carry look ahead adder consists of three levels of logic



- In the first level it generates all the P and G signals and it provides  $t_{pd}$  delay.
- In the second level the carry look ahead adder consists of four two ~~in~~ level implementation logic circuit that generates carry signal  $C_1, C_2, C_3$  and  $C_4$  as defined by the above expression & it provides  $2 t_{pd}$  time delay.
- In the third level four XOR gates which generates the sum  $(S_0, S_1, S_2, S_3)$  and it provides  $t_{pd}$  time delay. Therefore the total time delay by the adder is  $4 t_{pd}$

### Signed number Representation

- 2's representation is the best choice for addition of two signed numbers.
- If  $X$  &  $Y$  are the two positive numbers then
  - $X - Y = X + (-Y) = X + (2\text{'s comp}^n \text{ of } Y)$
  - $-X + Y = (2\text{'s comp}^n \text{ of } X) + Y$
  - $-X - Y = (2\text{'s comp}^n \text{ of } X) + (2\text{'s comp}^n \text{ of } Y)$

Find the value of the following expression by using 2's complement.  
 (i)  $55 + 27$   
 Consider the size of register to be 8 bits.

| Number | Binary          | 2's complement  |
|--------|-----------------|-----------------|
| 55     | 0 0 1 1 0 1 1 1 | 1 1 0 0 1 0 0 1 |
| 27     | 0 0 0 1 1 0 1 1 | 1 1 1 0 0 1 0 1 |

$$\begin{array}{r}
 00110111 \\
 00011011 \\
 \hline
 01000010
 \end{array}$$

(ii)  $-55 + 27$

$$\begin{array}{r}
 11001001 \\
 + 00011011 \\
 \hline
 11100100 \leftarrow -28
 \end{array}$$

2's comp  $\rightarrow$  0 0 0 1 1 0 0 0 (28)

(iii)  $55 - 27$

$$\begin{array}{r}
 00110111 \\
 11100101 \\
 \hline
 100011100
 \end{array}$$

↳ Discard

## Binary Multiplication (Unsigned number)

As compared to the addition and subtraction multiplication is a complex operation whether perform in a hardware or software.

• For multiplication of unsigned binary integers following points are considered -

1) Multiplication involves the generation of partial product, one for each digit in the multiplier.

2) These partial products are then summed to produce the final product.

3) The partial products are easily defined when the multiplier bit is zero. then partial product is zero.

4) The total product is produced by summing the partial products, for this operation each successful partial product is shifted one position to the left related to the preceding partial products.

<sup>Imp</sup> 5) The multiplication of two (n-bits) binary number (integer) produces a product of upto  $2n$  bits in length. therefore the product of the two 4 bit numbers fit in the eight bit length.

$$\begin{array}{r} 1011 \quad (\text{Multiplicand (M)}) \\ 0101 \quad (\text{Multiplier (Q)}) \\ \hline 01011 \\ 0000 \\ 1011 \\ 0000 \\ \hline 0110111 \end{array}$$

• In the binary system multiplication of the multiplicand by 1 bit at multiplier is easy.

• If the multiplier bit is 1, the multiplicand is entered in the appropriate position to be added to the partial product.



i)  $12 \times 19 = 1100 \times 1110$

| C | A    | Q    | M    | Operations                |
|---|------|------|------|---------------------------|
| 0 | 0000 | 1110 | 1100 | Initial value             |
| 0 | 0000 | 0111 | 1100 | $q_0 = 0$ (shift right)   |
| 0 | 1100 | 0111 | 1100 | $q_0 = 1$ then add with A |
| 0 | 0110 | 0011 | 1100 | shift right               |
| 1 | 0010 | 0011 | 1100 | $q_0 = 1$ then add with A |
| 0 | 1001 | 0001 | 1100 | shift right               |
| 1 | 0101 | 0001 | 1100 | $q_0 = 1$ then add with A |
| 0 | 1010 | 1000 | 1100 | shift right               |

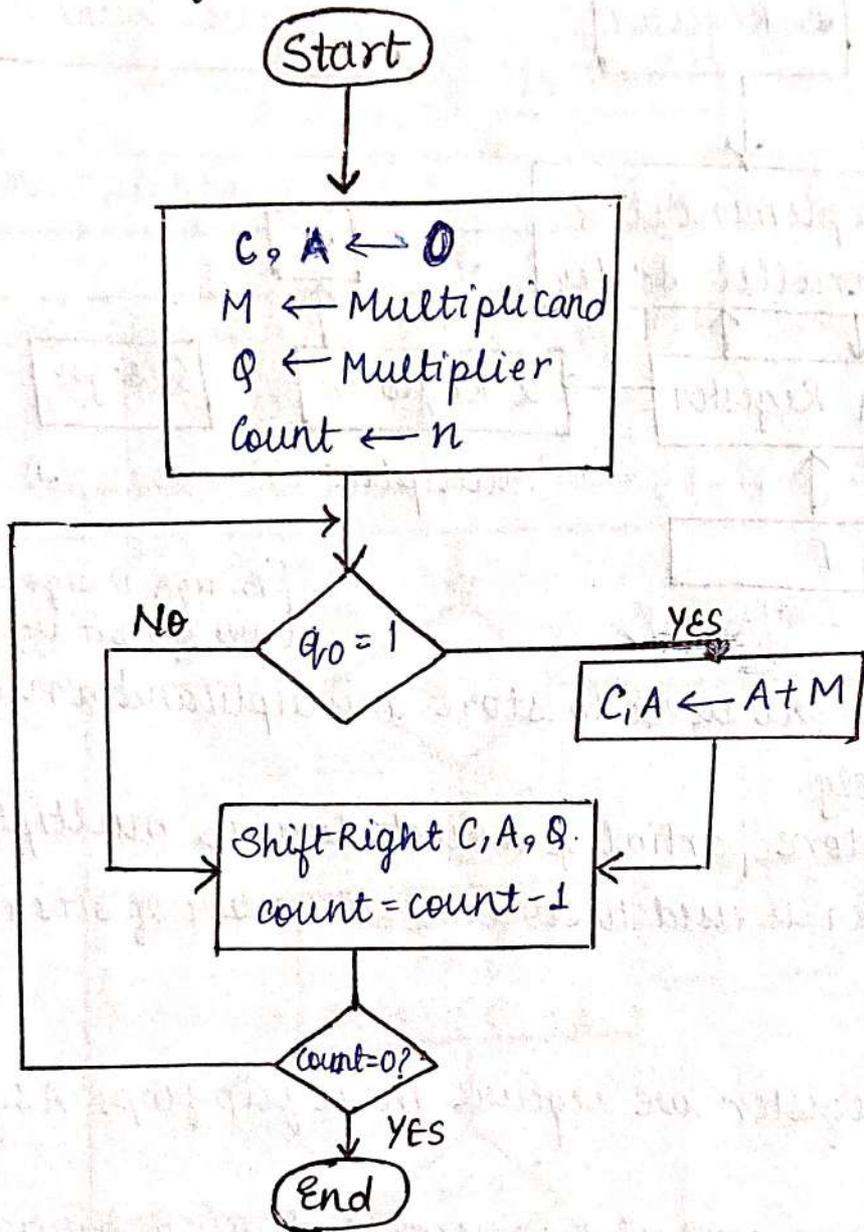
ii)  $9 \times 12 = 1001 \times 1100$

| C | A    | Q    | M    | Operations                |
|---|------|------|------|---------------------------|
| 0 | 0000 | 1100 | 1001 | Initial value             |
| 0 | 0000 | 0110 | 1001 | $q_0 = 0$ shift right     |
| 0 | 0000 | 0011 | 1001 | $q_0 = 0$ shift right     |
| 0 | 1001 | 0011 | 1001 | $q_0 = 1$ then add with A |
| 0 | 0100 | 1001 | 1001 | shift right               |
| 0 | 1101 | 1001 | 1001 | $q_0 = 1$ then add with A |
| 0 | 0110 | 1100 | 1001 | shift right               |

iii)  $19 \times 21 = 10011 \times 10101$

| C | A     | Q     | M     | Operations                 |
|---|-------|-------|-------|----------------------------|
| 0 | 00000 | 10101 | 10011 | Initial value              |
| 0 | 10011 | 10101 | 10011 | $q_0 = 1$ add with A       |
| 0 | 01001 | 11010 | 10011 | shift right                |
| 0 | 00100 | 11101 | 10011 | $q_0 = 0$ then shift right |
| 0 | 10111 | 11101 | 10011 | $q_0 = 1$ then add with A  |
| 0 | 01011 | 11110 | 10011 | shift right                |
| 0 | 00101 | 11111 | 10011 | $q_0 = 0$ shift right      |
| 0 | 11000 | 11111 | 10011 | $q_0 = 1$ then add with A  |
| 0 | 01100 | 01111 | 10011 | shift right                |

A flowchart of the Multiplication is as shown:-



Multiplication algorithm in Signed magnitude representation -

- The sign of the product is determined from the sign of the multiplicand and multiplier.
- If they are same, sign of the product is positive else negative. (based on XOR gates).

Hardware Implementation

The components require for the hardware implementation of the multiplication algorithm is given by

B sign

Multiplicand

B Register

Sequence Counter

Complemental & Parallel Adder

A sign

A Register

Q Register

Q sign

Q<sub>0</sub>

Multiplier

0 → E

1 bit storage element

[B sign, Q sign, A sign] are of 1 bit register

### Registers:-

- Two registers B and Q are used to store multiplicand and multiplier respectively.
- Register A is used to store partial product during multiplication.
- Sequence counter register is used to store the number of bits in the multiplier.

### Flip flops:-

- To store the sign bit register we require three flip flops A sign, B sign or Q sign.
- Flip flop E is used to store carry bit generated during partial product addition.

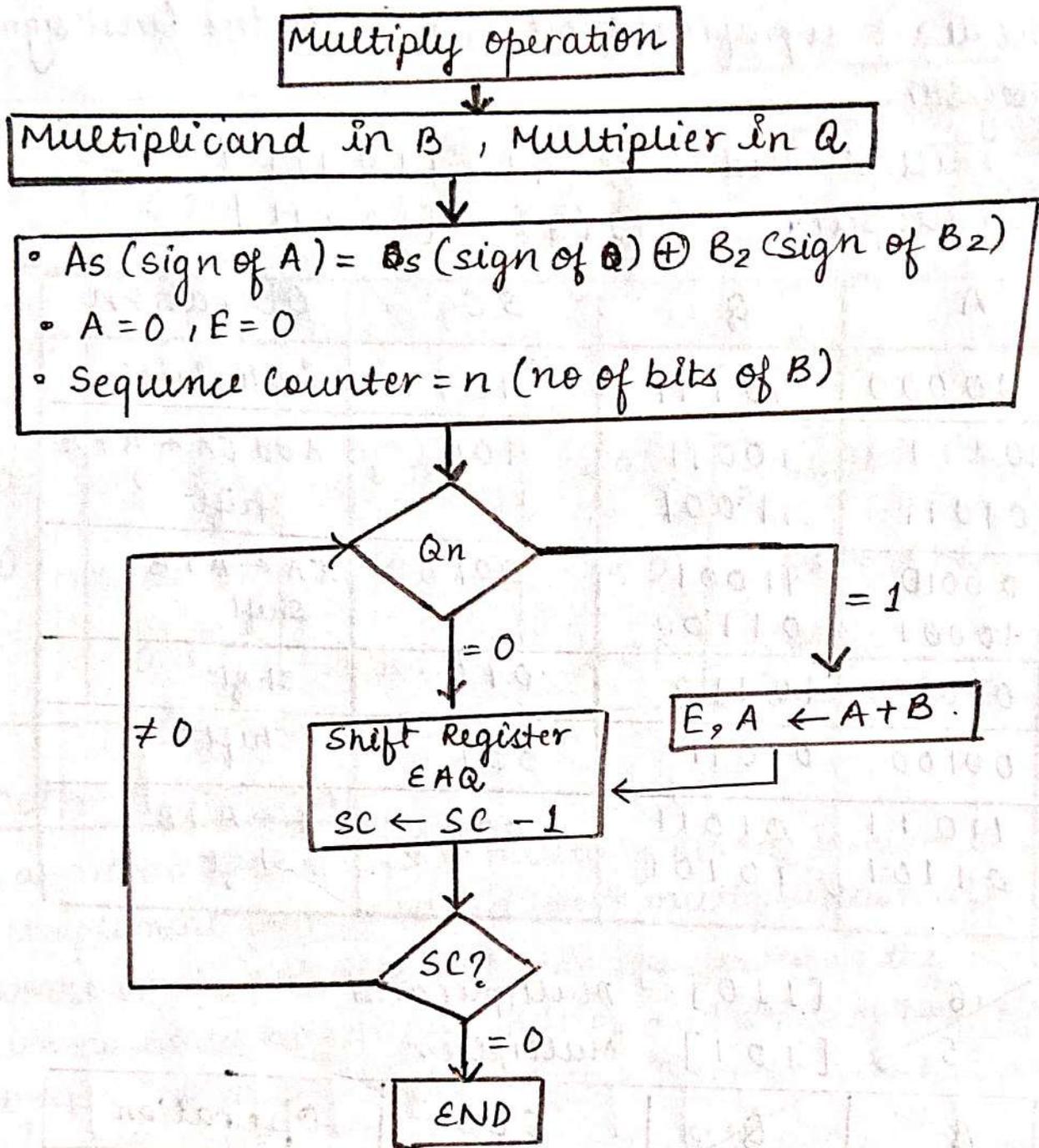
### Complement and parallel Adder

This hardware unit is used to calculate partial products. (This is it performs the required addition)

### Operation:-

- Initially multiplicand is stored in register B and the multiplier is stored in register Q.
- Sign of the B and Q are compared by using the XOR function & the output is stored in the A sign.
- Initially ~~0~~ 0 is assigned to register A and E flip flop.
- Sequence counter is initialized with value n. where n is the number of bits in the multiplicand.

# Flowchart for multiplication



- Now the LSB of the multiplier is checked.
- If it is 1 add the content of register A with B multiplicand and then result is assigned in register A with the carry bit in flip flop E.
- Now the content of the EAQ is shifted to the right by 1 position.
- If  $Q_n = 0$  then only shift right operation on the content of EAQ is performed and the content of sequence counter decremented by 1.
- Now check the content of sequence counter if it is 0 end the process and the final product is present in the register A & B.

else repeat the process.

• A hold the most significant bit & Q holds the least significant bit of the register.

Example:- Multiplicand = 23 (Q) [10111]  
 Multiplier = 19 (Q) [10011]

| E | A     | Q     | SC  | Operation                 |
|---|-------|-------|-----|---------------------------|
| 0 | 00000 | 10111 | 101 | Initial value             |
| 0 | 10111 | 10011 | 100 | Add $EA \leftarrow A + B$ |
| 0 | 01011 | 11001 |     | Shift                     |
| 1 | 00010 | 11001 | 001 | $EA \leftarrow A + B$     |
| 0 | 10001 | 01100 |     | Shift                     |
| 0 | 01000 | 10110 | 010 | Shift                     |
| 0 | 00100 | 01011 | 001 | Shift                     |
| 0 | 11011 | 01011 | 000 | $A \leftarrow A + B$      |
| 0 | 01101 | 10101 |     | Shift                     |

$$6 \times 5 = 30 = [11010]$$

$$5 = [101]$$

| E | A   | Q   | SC  | Operation                    |
|---|-----|-----|-----|------------------------------|
| 0 | 000 | 101 | 011 | Initial value                |
| 0 | 110 | 101 | 010 | $EA \leftarrow A+B$<br>Shift |
| 0 | 011 | 010 |     | Shift                        |
| 0 | 001 | 001 | 001 | Shift                        |
| 0 | 111 | 101 | 000 | $EA \leftarrow A+B$<br>Shift |
|   | 011 | 110 | 000 |                              |

## Booth Algorithm

The algorithm that is used to multiply binary integer in signed 2's complement form is called booth multiplication algorithm.

It works on the principle that string of zeroes in the multiplier require no addition but just shifting and string of one 1's in the multiplier from bit weight  $2^k$  to  $2^m$  can be treated as  $(2^{k+1} - 2^m)$

For Example:-  $0 \overset{k=4}{111} \overset{m=2}{00}$

$$(2^{k+1} - 2^m)$$

$$(2^5 - 2^2)$$

$$= 32 - 4 = 28$$

Therefore the multiplication  $M \times 28$ , where  $M$  is the multiplicand and 28 is the multiplier that can be done by  $M \times (2^5 - 2^2)$

$$\Rightarrow (M \times 2^5) - (M \times 2^2)$$

Therefore the product can be obtained by shifting the binary multiplicand ( $M$ ) five times to the left and subtracting  $m$  shifted left two times

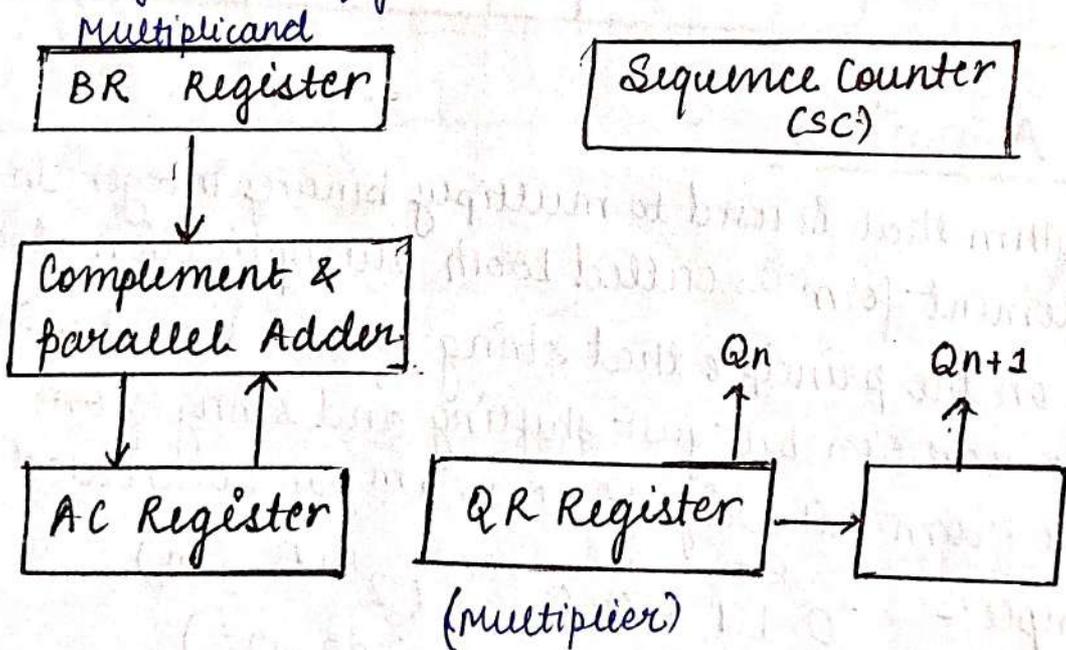
Booth algorithm requires the examination of the multiplier bits and shifting of the partial product prior to the shifting

the multiplicand may be added to the partial product subtracted from the partial product or left unchanged according to the following rules:-

- 1) The partial product does not change when the multiplier bit is identical to the previous multiplier bit.
- 2) The multiplicand is subtracted from the partial product upon encountering the first <sup>or</sup> least significant 1 in the string of 1's in the multiplier.
- 3) The multiplicand is added to the partial product upon encountering the first zero (provided that there was a previous one) is a string of zeroes in the multiplier.

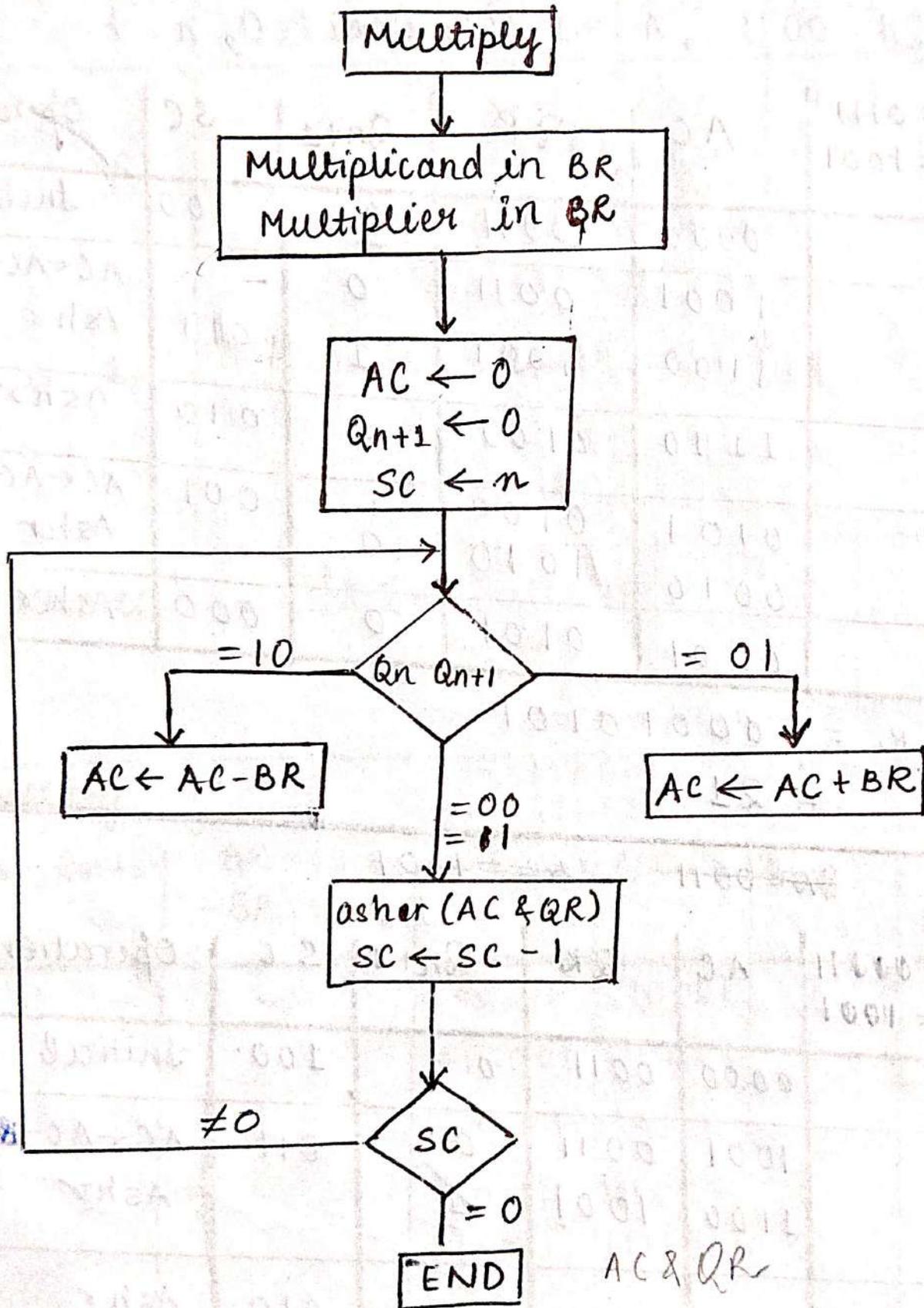
00/11 → unchanged  
 10 → Subt  
 01 → Add

The Hardware implementation of the booth alg. requires the register configuration as shown:-



An extra flip flop  $Q_{n+1}$  is include to QR for the double bit inscription of the multiplier.

# Flowchart of Booth Algorithm



Q)  $7 \times 3$ .

$BR = 0111$ ,  $QR = 0011$ ,  $AC = 0000$ ,  $Q_{n+1} = 0$ ,  $n = 4$

| $Q_n$ | $Q_{n+1}$ | $BR = 0111$<br>$-BR = 1001$ | AC                | QR           | $Q_{n+1}$ | SC  | Operation                       |
|-------|-----------|-----------------------------|-------------------|--------------|-----------|-----|---------------------------------|
|       |           |                             | 0000              | 0011         | 0         | 100 | Initial                         |
| 1     | 0         |                             | 1001<br>↓<br>1100 | 0011<br>1001 | 0<br>1    | 011 | $AC \leftarrow AC - BR$<br>Ashr |
| 1     | 1         |                             | 1110              | 0100         | 1         | 010 | Ashr                            |
| 0     | 1         |                             | 0101<br>0010      | 0100<br>1000 | 1<br>0    | 001 | $AC \leftarrow AC + BR$<br>Ashr |
| 00    | 0         |                             | 0001              | 0101         | 0         | 000 | Ashr                            |

Result = AC & QR = 00010101  
= 21

Multiply  $+7 \times 3$ .

~~QR = 0011~~

~~BR = 1001~~

| $Q_n$ | $Q_{n+1}$ | $BR = 0111$<br>$-BR = 1001$ | AC                | QR           | $Q_{n+1}$ | SC  | Operation                       |
|-------|-----------|-----------------------------|-------------------|--------------|-----------|-----|---------------------------------|
|       |           |                             | 0000              | 0011         | 0         | 100 | Initial                         |
| 1     | 0         |                             | 1001<br>1100      | 0011<br>1001 | 0<br>1    | 011 | $AC \leftarrow AC - BR$<br>Ashr |
| 1     | 1         |                             | 1110              | 0100         | 1         | 010 | Ashr                            |
| 0     | 1         |                             | 0011<br>↓<br>0010 | 0100<br>1010 | 1<br>0    | 001 | $AC \leftarrow AC + BR$<br>Ashr |
| 0     | 0         |                             | 0001              | 0101         | 0         | 000 | Ashr                            |

Multiply  $-7 \times 3$  BR = 1001

| $Q_n$ | $Q_{n+1}$ | BR = 1001<br>-BR = 0111 | AC   | QR   | $Q_{n+1}$ | SC  | Operation    |
|-------|-----------|-------------------------|------|------|-----------|-----|--------------|
|       |           |                         | 0000 | 0011 | 0         | 100 | Initial      |
| 1     | 0         |                         | 0111 | 0011 | 0         | 011 | AC ← AC - BR |
|       |           |                         | 0011 | 1001 | 1         |     | Ashx         |
| 1     | 1         |                         | 0001 | 1100 | 1         | 010 | Ashx         |
| 0     | 1         |                         | 1010 | 1100 | 1         | 001 | AC ← AC + BR |
|       |           |                         | 1101 | 0110 | 0         |     | Ashx         |
| 0     | 0         |                         | 1110 | 1011 | 0         | 000 | Ashx         |

Result = AC & QR = 11101011  
= -21

↓  
2's complement ⇒ 00010101 ⇒ 21

Multiply  $-17 \times -15$   
BR x QR

17 = 010001      15 = 001111  
-17 = 101101      -15 = 110001

| $Q_n$ | $Q_{n+1}$ | BR = 101111<br>-BR = 010001 | AC     | QR     | $Q_{n+1}$ | SC  | Operation    |
|-------|-----------|-----------------------------|--------|--------|-----------|-----|--------------|
|       |           |                             | 000000 | 110001 | 0         | 110 | Initial      |
| 1     | 0         |                             | 010001 | 110001 | 0         | 101 | AC ← AC - BR |
|       |           |                             | 001000 | 111000 | 1         |     | Ashx         |
| 0     | 1         |                             | 110111 | 111000 | 1         | 100 | AC ← AC + BR |
|       |           |                             | 111011 | 111100 | 0         |     | Ashx         |
| 0     | 0         |                             | 111101 | 111110 | 0         | 011 | Ashx         |
| 0     | 0         |                             | 111110 | 111111 | 0         | 010 | Ashx         |
| 1     | 0         |                             | 001111 | 111111 | 0         | 001 | AC ← AC - BR |
|       |           |                             | 000111 | 111111 | 1         |     | Ashx         |
| 1     | 1         |                             | 000011 | 111111 | 1         | 000 | Ashx         |

Result → 000011111111

Solve  $(+15 \times -13)$  using Booth's Algor.

15 = 01111  
 -15 = 1'0001  
 13 = 01101  
 -13 = 10011

| $Q_n$ | $Q_{n+1}$ | BR = 01111<br>-BR = 10001 | AC    | QR    | $Q_{n+1}$ | SC  | operation           |
|-------|-----------|---------------------------|-------|-------|-----------|-----|---------------------|
|       |           |                           | 00000 | 10011 | 0         | 101 | Initial             |
| 1     | 0         |                           | 10001 | 10011 | 0         | 000 | AC ← AC - BR<br>Ash |
|       |           |                           | 10000 | 11001 | 1         |     |                     |
| 1     | 1         |                           | 01100 | 01100 | 1         | 011 | Ash                 |
| 0     | 1         |                           | 01011 | 01100 | 0         | 010 | AC ← AC + BR<br>Ash |
|       |           |                           | 00101 | 10110 | 0         |     |                     |
| 0     | 0         |                           | 00010 | 11101 | 0         | 001 | Ash                 |
| 1     | 0         |                           | 10011 | 11011 | 0         | 000 | AC ← AC - BR        |
|       |           |                           | 11001 | 11101 | 1         |     |                     |

Result → AC & QR = 110011101

-15 x -12  
 15 = 01111  
 -15 = 10001  
 12 = 01100  
 -12 = 10100

| $Q_n$ | $Q_{n+1}$ | BR = 10001<br>-BR = 01111 | AC    | QR    | $Q_{n+1}$ | SC  | operation    |
|-------|-----------|---------------------------|-------|-------|-----------|-----|--------------|
|       |           |                           | 00000 | 10100 | 0         | 101 | Initial      |
| 0     | 0         |                           | 00000 | 01010 | 0         | 100 | Ash          |
| 0     | 0         |                           | 00000 | 00101 | 0         | 011 | Ash          |
| 1     | 0         |                           | 01111 | 00101 | 0         | 010 | AC ← AC - BR |
|       |           |                           | 00111 | 10010 | 1         |     | Ash          |
| 0     | 1         |                           | 11000 | 10010 | 1         | 001 | AC ← AC + BR |
|       |           |                           | 11100 | 01001 | 0         |     | Ash          |
| 1     | 0         |                           | 01011 | 01001 | 0         | 000 | AC ← AC - BR |
|       |           |                           | 00101 | 10100 | 1         |     | Ash          |

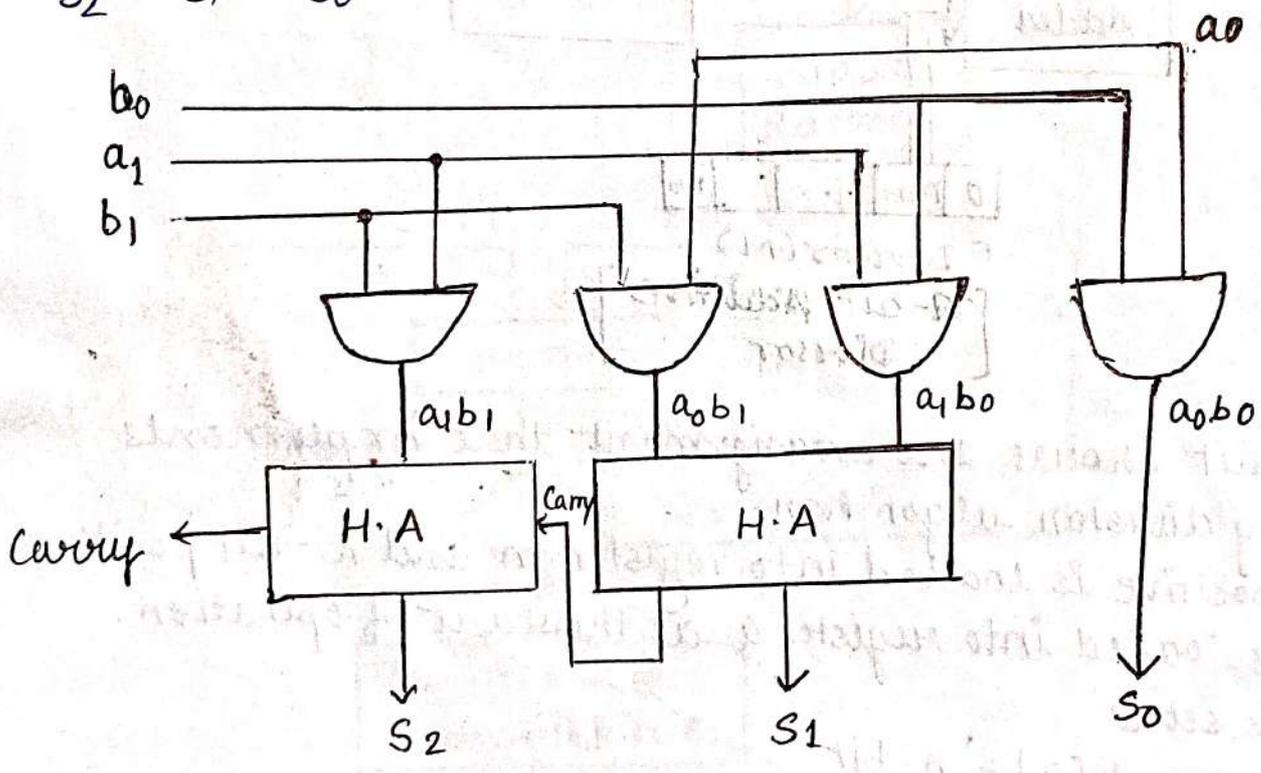
Result :- 0010110100

Array Multiplier An array multiplier is a digital circuit (combinational circuit) used for multiplying two binary numbers by inclined an array of full adders and half adders. This array is used for simultaneous addition for various product term in word.

• To form the various product term an array of AND gate is used before the adder carrier.

• Consider the multiplication of two 2 bit numbers as shown -

$$\begin{array}{r}
 a_1 \quad a_0 \\
 b_1 \quad b_0 \\
 \hline
 a_1 b_0 \quad a_0 b_0 \\
 a_0 b_1 \quad a_0 b_1 \\
 \hline
 S_2 \quad S_1 \quad S_0
 \end{array}$$



Division an algorithm which deals two integers A and B computes their remainder and quotient is called the division algorithm.

• Division algorithm are of two types :-

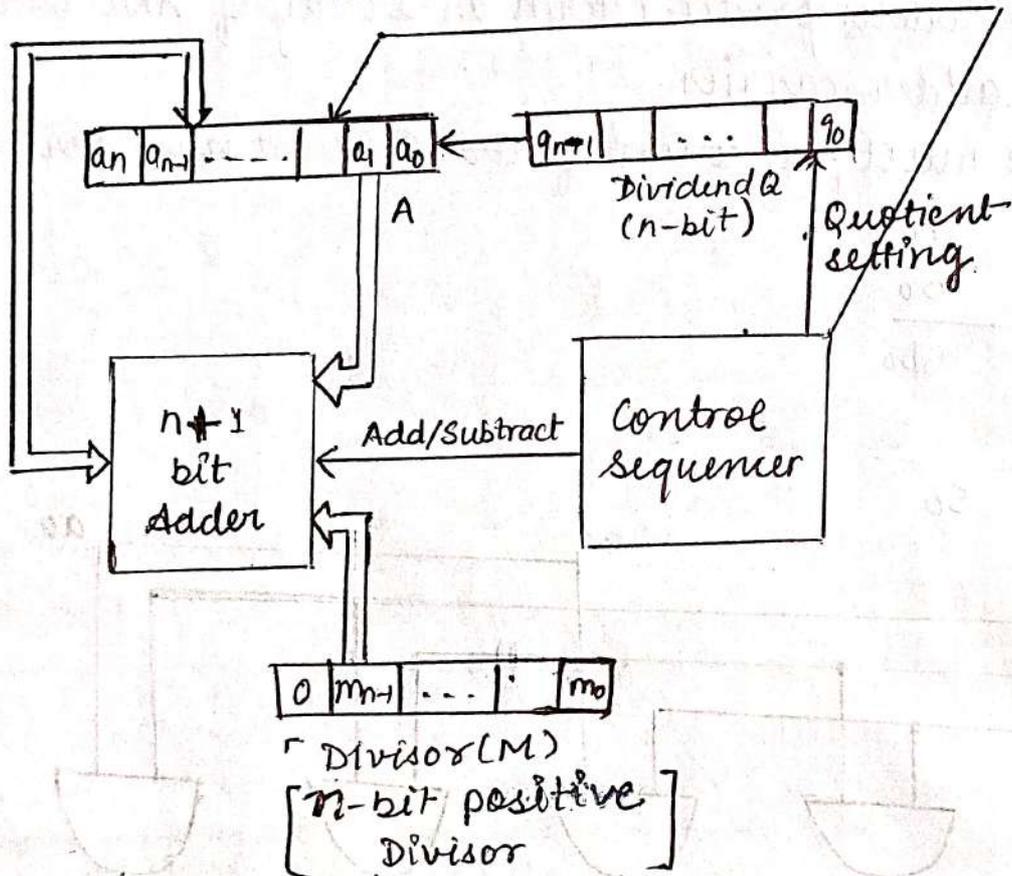
- 1) Slow algorithm
  - a) Restoring algorithm
  - b) Non-restoring algorithm

- 2) Fast algorithm
  - a) Newton - Raphson Algo.
  - b) SRT division

## Restoring division Algo.

Restoring division Algo is a traditional algo for performing binary division.

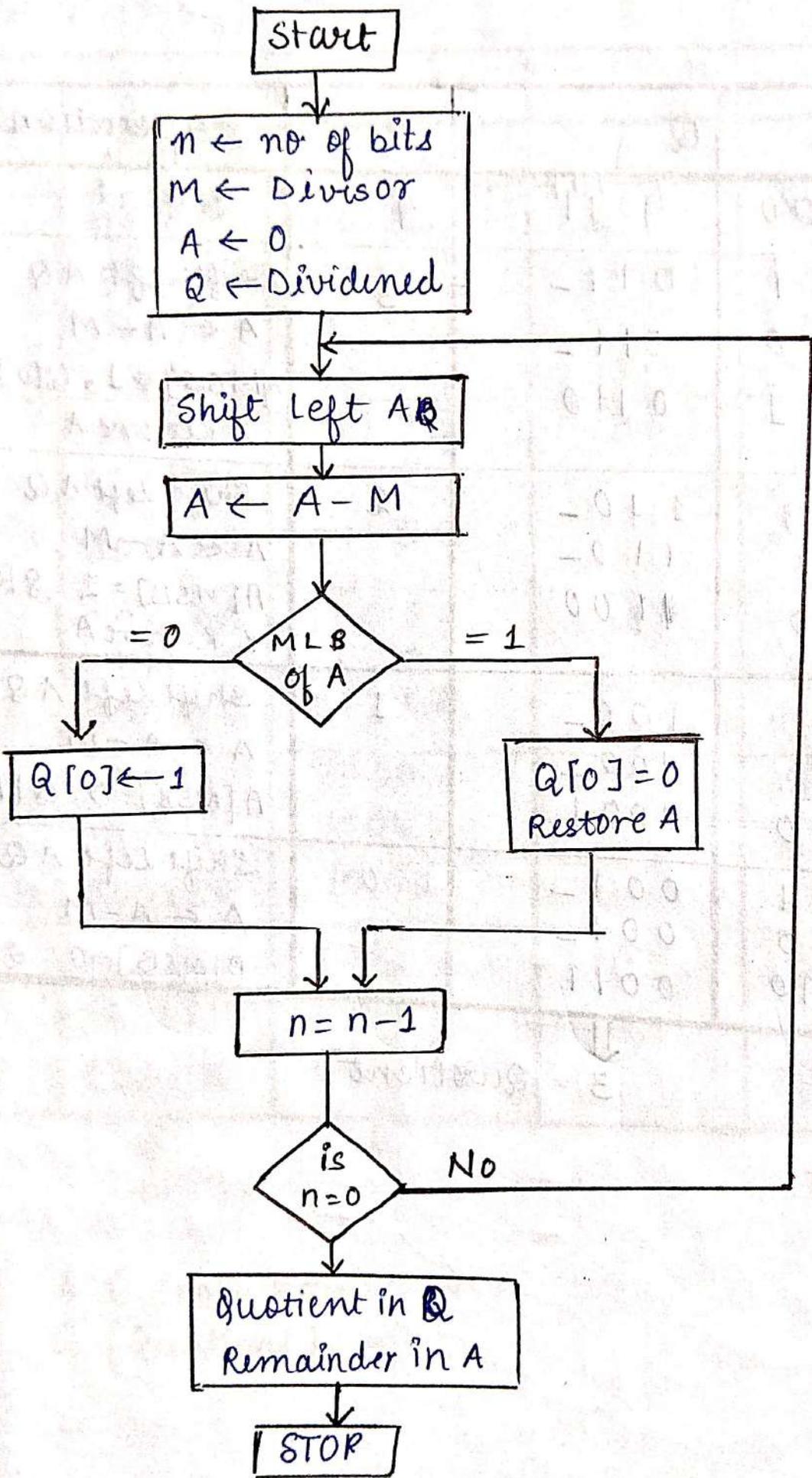
- In restoring division we restore the partial remainder after each subtraction if the result is negative.



logical

Above circuit shows the arrangement that implements the restoring division algorithm.

- An  $n$ -bit positive is loaded into register  $m$  and  $n$ -bit positive dividend is loaded into register  $q$  at the start of operation.
- Register  $A$  is set to
  - Dividend  $[Q] = n$ -bit
  - Divisor  $[M] = n+1$  bit
  - $A = n+1$  [initially reset]



Q) Divide 11 by 3

| M     | A                       | Q                    | n | operation  |
|-------|-------------------------|----------------------|---|--|
| 00011 | 00000                   | 1011                 | 4 | Initial  |
|       | 00001<br>11110<br>00001 | 011-<br>011-<br>0110 | 3 | Shift left A Q<br>A ← A - M<br>A[MSB] ≠ 1, Q[0] = 0<br>& Restore A |
|       | 00010<br>11111<br>00010 | 110-<br>110-<br>100  | 2 | Shift left A Q<br>A ← A - M<br>A[MSB] = 1, Q[0] = 0<br>& Restore A |
|       | 00101<br>00010<br>00010 | 100-<br>100-<br>1001 | 1 | Shift left A Q<br>A ← A - M<br>A[MSB] = 0, Q[0] = 1                |
|       | 00101<br>00010<br>00010 | 001-<br>001-<br>0011 | 0 | Shift left A Q<br>A ← A - M<br>A[MSB] = 0, Q[0] = 1                |

Remainder = 2

3 = Quotient

Q) Divide 19 by 5

| M      | A                          | Q                       | n | Operation  |
|--------|----------------------------|-------------------------|---|--|
| 000101 | 000000                     | 10011                   | 5 | Initial  |
|        | 000001<br>111100<br>000001 | 0011-<br>0011<br>00110  | 4 | Shift left A Q<br>A ← A - M<br>A[MSB] = 1, Q[0] = 0<br>and Restore A |
|        | 000000<br>111101<br>000010 | 0110-<br>0110-<br>01100 | 3 | Shift left A Q<br>A ← A - M<br>A[MSB] = 1, Q[0] = 0<br>Restore A     |
|        | 000100<br>111111<br>000100 | 1100-<br>1100-<br>11000 | 2 | Shift left A Q<br>A ← A - M<br>A[MSB] = 1, Q[0] = 0<br>Restore A     |
|        | 001001<br>000100<br>000100 | 1000-<br>1000-<br>10001 | 1 | Shift left A Q<br>A ← A - M<br>A[MSB] = 0, Q[0] = 1                  |
|        | 001001<br>000000<br>000100 | 0000-<br>0000-<br>00011 | 0 | Shift left A Q<br>A ← A - M<br>A[MSB] = 0, Q[0] = 1                  |

↓  
4

↓  
3

A (Remainder) = 4  
Q (Quotient) = 3

Divide 55 by 13

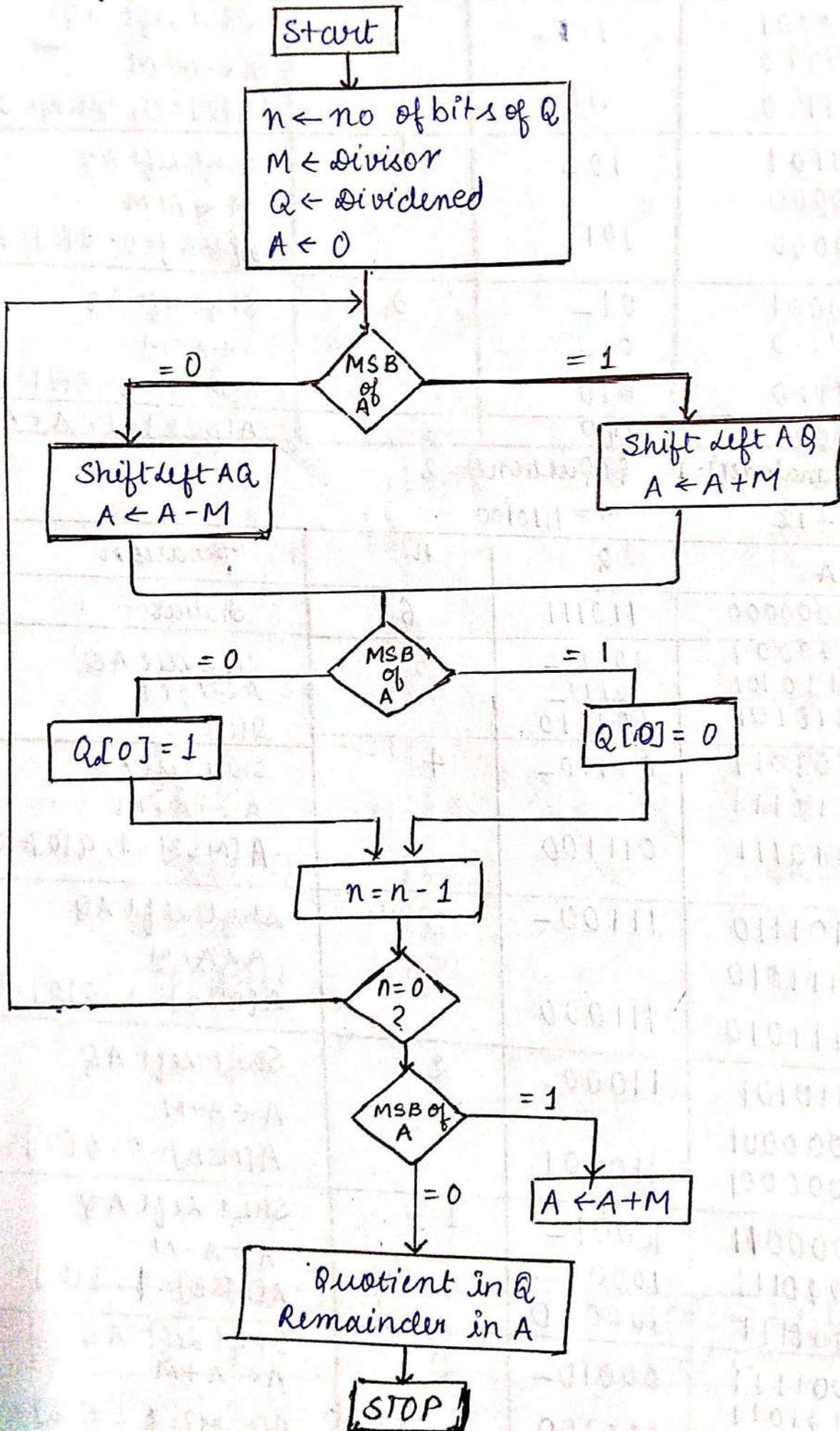
| M       | A                             | Q                          | n | Operation  |
|---------|-------------------------------|----------------------------|---|--|
| 1110011 |                               |                            |   |  |
| 001101  | 0000000                       | 110111                     | 6 | Initial  |
|         | 0000001<br>1110100<br>0000001 | 10111-<br>10111-<br>101110 | 5 | Shift left AQ.<br>$A \leftarrow A - M$<br>$A[\text{MSB}] = 1, Q[0] = 0$<br>Restore A |
|         | 0000010<br>1110110<br>0000011 | 01110-<br>01110-<br>011100 | 4 | Shift left AQ<br>$A \leftarrow A - M$<br>$A[\text{MSB}] = 1, Q[0] = 0$<br>Restore A  |
|         | 0000110<br>1111001<br>0000110 | 11100-<br>11100-<br>111000 | 3 | Shift left AQ.<br>$A \leftarrow A - M$<br>$A[\text{MSB}] = 1, Q[0] = 0$<br>Restore A |
|         | 0001101<br>0000000<br>0000000 | 11000-<br>11000-<br>110001 | 2 | Shift left AQ.<br>$A \leftarrow A - M$<br>$A[\text{MSB}] = 0, Q[0] = 1$              |
|         | 0000001<br>1110100<br>0000001 | 10001-<br>10001-<br>100010 | 1 | Shift left AQ<br>$A \leftarrow A - M$<br>$A[\text{MSB}] = 1, Q[0] = 0$<br>Restore A  |
|         | 0000011<br>1110110<br>0000011 | 00010-<br>00010-<br>000100 | 0 | Shift left AQ<br>$A \leftarrow A - M$<br>$A[\text{MSB}] = 1, Q[0] = 0$<br>Restore A  |

Remainder(A) = 3

4 = Quotient (Q)

# Non Restoring

## Flowchart of unsigned integer division by using Non restoring method



1) Divide 7 by 3

-M = 1101

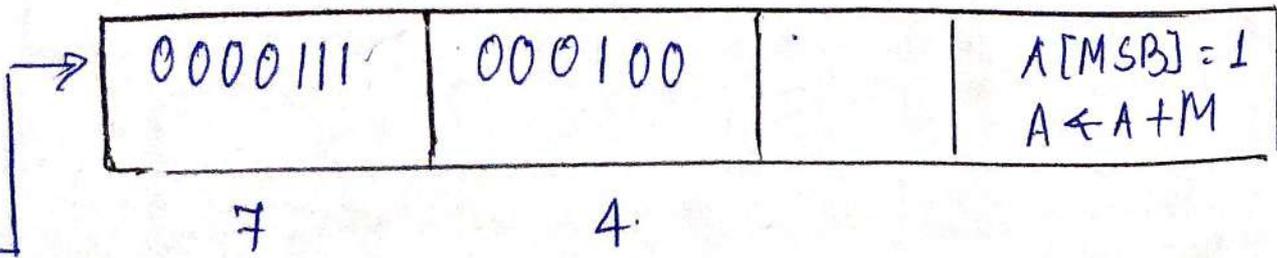
| M    | A                    | Q                   | n | Operation  |
|------|----------------------|---------------------|---|--|
| 0011 | 0000                 | 111                 | 3 | Initial  |
|      | 0001<br>1110<br>1110 | 11 -<br>.<br>110    | 2 | Shift left AQ<br>A ← A - M<br>Q[0] = 0, A[MSB] = 1 |
|      | 1101<br>0000<br>0000 | 10 -<br><br>101     | 1 | Shift left AQ<br>A ← A + M<br>A[MSB] = 0, Q[0] = 1 |
|      | 0001<br>1110<br>1110 | 01 -<br>01 -<br>010 | 0 | Shift left AQ<br>A ← A - M<br>A[MSB] = 1, Q[0] = 0 |
|      | 0001                 | 010                 |   | A[MSB] = 1, A ← A + M                              |

A (Remainder) = 1      Q (Quotient) = 2

2) Divide 55 by 12

-M = 1110100

| M       | A                             | Q                            | n | Operation  |
|---------|-------------------------------|------------------------------|---|--|
| 0001100 | 0000000                       | 110111                       | 6 | Initial  |
|         | 000001<br>1110101<br>1110101  | 10111 -<br>10111 -<br>101110 | 5 | Shift left AQ<br>A ← A - M<br>Q[0] = 0             |
|         | 1101011<br>1110111<br>1110111 | 01110 -<br><br>011100        | 4 | Shift left AQ<br>A ← A + M<br>A[MSB] = 1, Q[0] = 0 |
|         | 1101110<br>1111010<br>1111010 | 11100 -<br><br>111000        | 3 | Shift left AQ<br>A ← A + M<br>A[MSB] = 1, Q[0] = 0 |
|         | 1110101<br>0000001<br>0000001 | 11000 -<br><br>110001        | 2 | Shift left AQ<br>A ← A + M<br>A[MSB] = 0, Q[0] = 1 |
|         | 0000001<br>1110111<br>1110111 | 10001 -<br>10001 -<br>100010 | 1 | Shift left AQ<br>A ← A - M<br>A[MSB] = 1, Q[0] = 0 |
|         | 1101111<br>1111011<br>1111011 | 00010 -<br><br>000100        | 0 | Shift left AQ<br>A ← A + M<br>A[MSB] = 1, Q[0] = 0 |



Q=4 R=-

## Floating point representation

We can represent the floating point numbers in the form of  $\pm S \times B^{\pm E}$

where,  $S$  = significant or Mantissa

$E$  = Exponent

$B$  = Is the base (for binary  $B = 2$ )

$I$  = Sign bit

→ The base (binary number), it is same for all numbers and need not be stored -

• It is assumed that radix point is to the right of the left most or most significant bit of the significant.

• There is one bit to the left of the radix point.

• Any floating point number can be represented in many ways -

a)  $0.110 \times 2^5$

b)  $110 \times 2^2$

c)  $0.0110 \times 2^8$

IEEE 754 format :-

1) Single precision (32 bit)

2) Double precision (64 bit)

| IEEE 754 format               | Sign bit | Biased exponent Bias | Bias | Fraction bits |
|-------------------------------|----------|----------------------|------|---------------|
| Single bit precision (32 bit) | 1        | 8                    | 127  | 23 bits       |
| Double bit precision (64 bit) | 1        | 11                   | 1023 | 52 bits       |

|          |               |                           |
|----------|---------------|---------------------------|
| Sign bit | Exponent bits | Fraction bits or mantissa |
|----------|---------------|---------------------------|

To simplify the operation on floating point number they are non-zero.

- A normalised number is one in which the most significant digit of the significant is non zero.
- For base-2 representation a normalised number is therefore one in its MSB of the significant is one. Therefore a normalised non-zero no. is in the form of  $\pm 1. b b b b \dots b \times 2^{\pm E}$  where  $b$  is either binary digit (one or zero).
- Because MSB is always one it is unnecessary to store this bit rather it is implicit.
- If the given no. is not normalise the no. maybe normalise by shifting the radix point to the right of the left most one bit and adjusting the exponent accordingly.

### Biasing:-

- Exponent is stored in the biased representation.
- In single precision 8 bit field produces the number. ( $2^8 = 256$   
(0-255))
- With the bias of 127 the true exponent values are in the range of -127 to +128.
- In double precision is 11 bit field produces the numbers from (0-2047)
- With the bias 1023 the true exponent values in the range -1023 to +1024.

### Working

- Normalization
- Storage
- Interpretation

Q) Represent the following no. by using single precision & double precision.  $(-0.00000100011001101)_2$

sol) 1) Normalization

$$1.00011001101 \times 2^{-6}$$

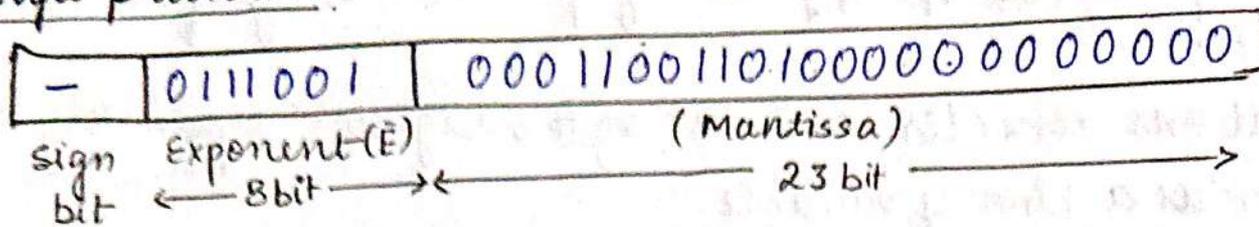
2) Storage (single precision):-

$$\text{Bias exponent } (\bar{E}) = E + \text{Bias} \\ = -6 + 127 = 121$$

$$= 121$$

$$= 0111001$$

single precision



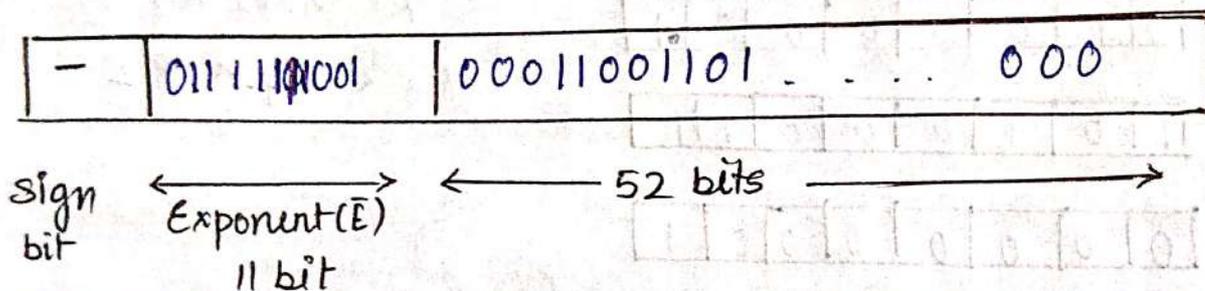
2) Double Precision

$$\text{Biased Exponent } (\bar{E}) = E + \text{Bias}$$

$$= -6 + 1023$$

$$= 1017$$

$$= 01111111001$$



1) Represent  $(-1234.125)_{10}$  into single precision and double precision.

$$10011010010.001$$

$$\begin{aligned} 0.125 \times 2 &= 0.250 \\ 0.250 \times 2 &= 0.500 \\ 0.500 \times 2 &= 1.000 \end{aligned}$$

1) Normalization:-

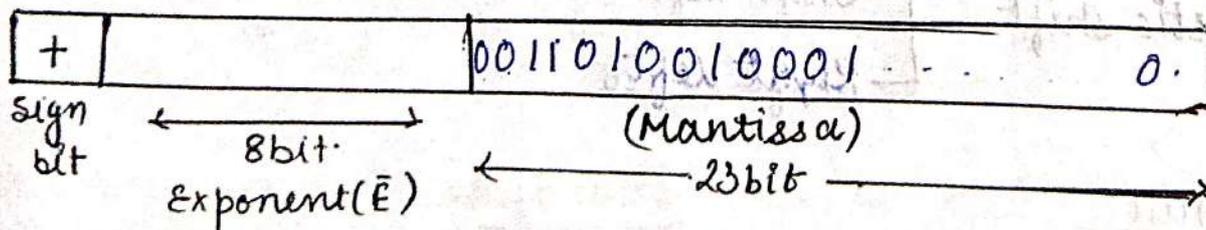
$$1.0011010010001 \times 2^{10}$$

2) single precision:-

$$\text{Bias Exponent } (\bar{E}) = E + \text{Bias}$$

$$= 10 + 127 = 137$$

$$=$$

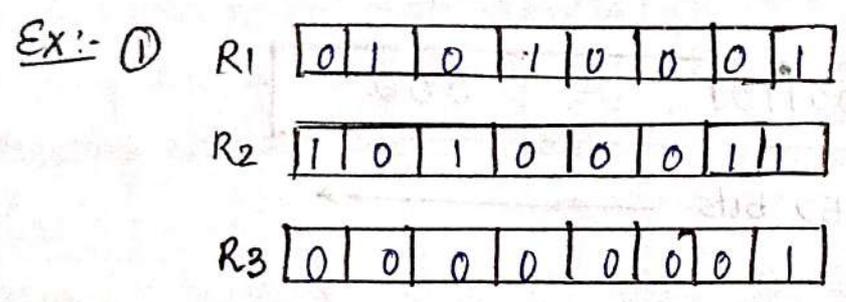


# Logic micro-operations

Logic micro-operations specify binary operation string of bit stored in a register

These operations consider each bit of the register separately & treat them as a binary variable.

- for Ex:-
- |                    |        |
|--------------------|--------|
| ① AND ( $\wedge$ ) | ⑤ NAND |
| ② OR ( $\vee$ )    | ⑥ XOR  |
| ③ NOT ( $-$ )      | ⑦ XNOR |
| ④ NOR              |        |



R1 AND R2  
 $R_3 \leftarrow R_1 \wedge R_2$

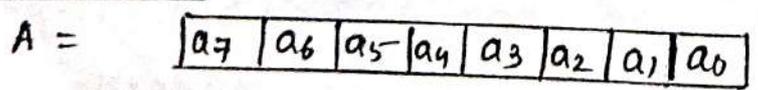
## Shift micro operations

• Shift micro operations are used for serial transfer of the data.

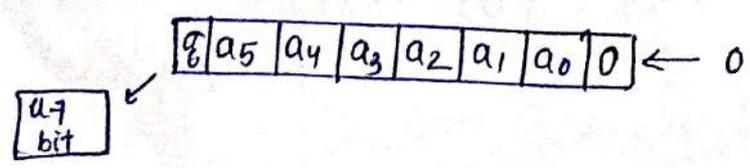
• Types of shift operations

- |                               |   |   |
|-------------------------------|---|---|
| ① Logical shift               | { | Shift left<br>Shift right                   |
| ② Circular shift<br>or Rotate | { | Circular shift left<br>Circular shift right |
| ③ Arithmetic shift            | { | Shift left<br>Shift right                   |

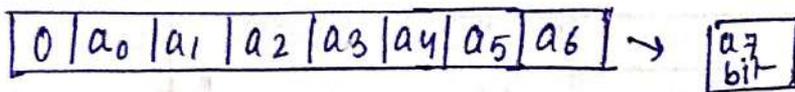
### 1) Logical shift



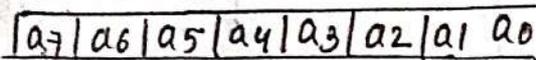
a) left shift



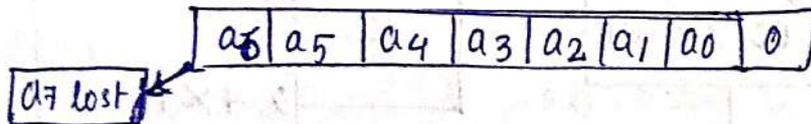
Shift Right



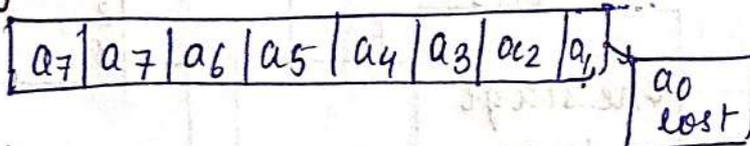
3) Arithmetic shift



a) Shift left



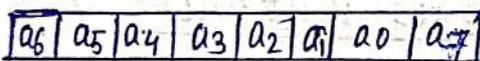
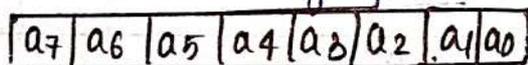
b) Shift right



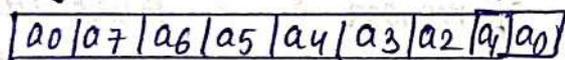
Circular Shift

In the circular shift the circulation of the bit of the register around the two ends without the loss of information

• Circular shift left



• Circular right



ALU DESIGN

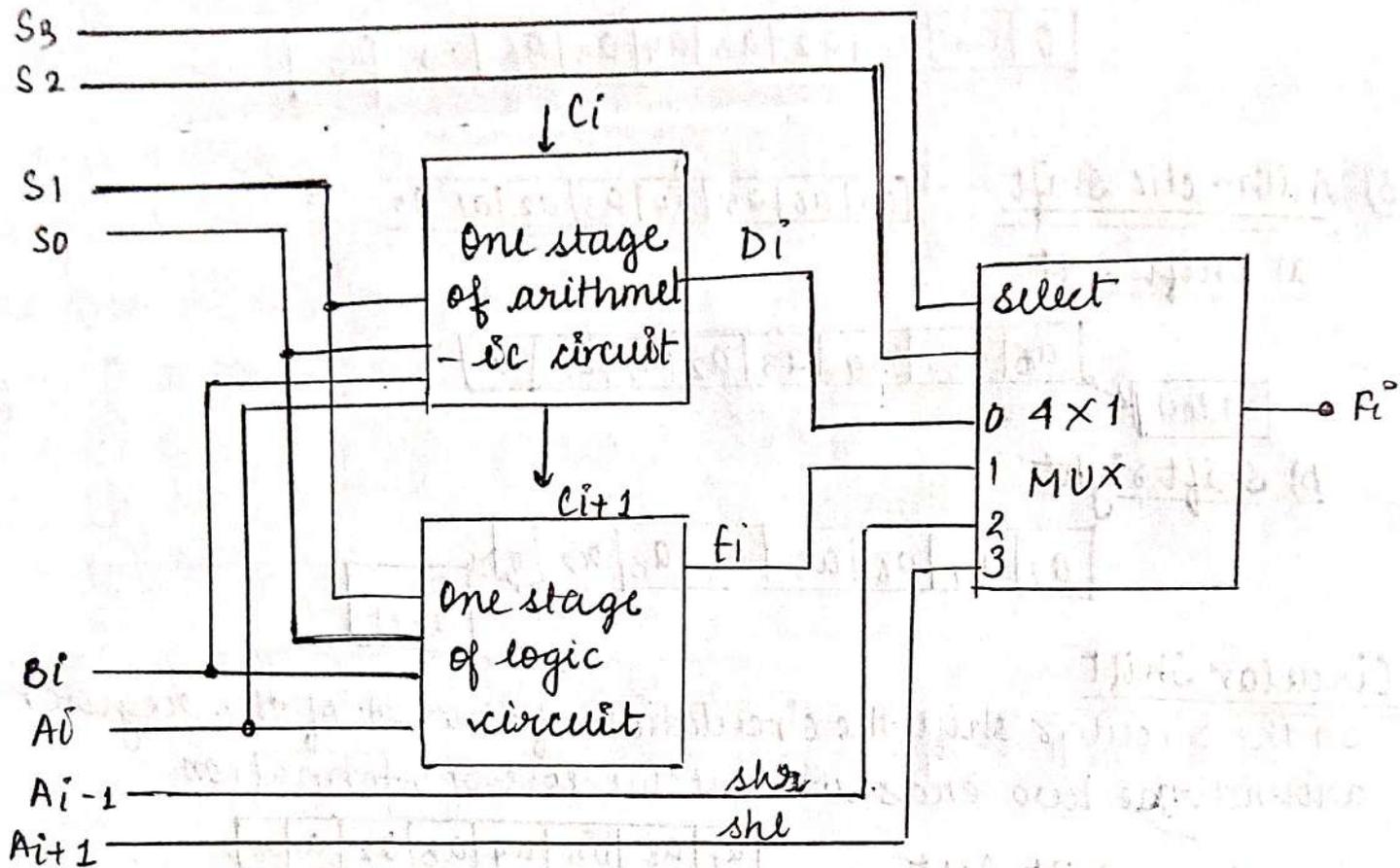
CPU contains CU, ALU, registers. ALU is responsible for arithmetic & logical operations basically ALU is a digital circuit that performs arithmetic operations like addition, subtraction, multiplication, division and logical operations like AND, OR, NOT

Types of ALU

1) Combinational ALU

2) Sequential ALU

Arithmetic and logic shift unit:- The ALU is a combinational circuit is that the entire register transfer operation from the source register through the ALU and into the destination register can be performed during one clock pulse period.



one stage of arithmetic logic shift unit

Operation select

| $S_3$ | $S_2$ | $S_1$ | $S_0$ | $C_{in}$ | Operation             | Function             |
|-------|-------|-------|-------|----------|-----------------------|----------------------|
| 0     | 0     | 0     | 0     | 0        | $F = A$               | Transfer A           |
| 0     | 0     | 0     | 0     | 1        | $F = A + 1$           | Increment A          |
| 0     | 0     | 0     | 1     | 0        | $F = A + B$           | Addition             |
| 0     | 0     | 0     | 1     | 1        | $F = A + B + 1$       | Add with carry       |
| 0     | 0     | 1     | 0     | 0        | $F = A + \bar{B}$     | subtract with borrow |
| 0     | 0     | 1     | 0     | 1        | $F = A + \bar{B} + 1$ | Subtraction          |
| 0     | 0     | 1     | 1     | 0        | $F = A - 1$           | Decrement A          |
| 0     | 0     | 1     | 1     | 1        | $F = A$               | Transfer A           |
| 0     | 1     | 0     | 0     | X        | $F = A \wedge B$      | AND                  |
| 0     | 1     | 0     | 1     | X        | $F = A \vee B$        | OR                   |
| 0     | 1     | 1     | 0     | X        | $F = A \oplus B$      | XOR                  |
| 0     | 1     | 1     | 1     | X        | $F = \bar{A}$         | Complement A         |
| 1     | 0     | X     | X     | X        | $F = shr A$           | Shift right A into F |
| 1     | 1     | X     | X     | X        | $F = shl A$           | Shift left A into F  |

# Sequential Logic Circuit based ALU:-

Two registers X and Y stores data on operands.

Select/control selects the appropriate arithmetic or logic operation which is performed over the data stored on register X and Y.

After execution of operation, the result will be stored in result register.

After execution of operation flags may be set such as carry, zero result, positive or negative result, overflow, division by zero etc.

