UNIT – 03 (III)

Interference of light :-

for ~~constant~~ constructive Interference.

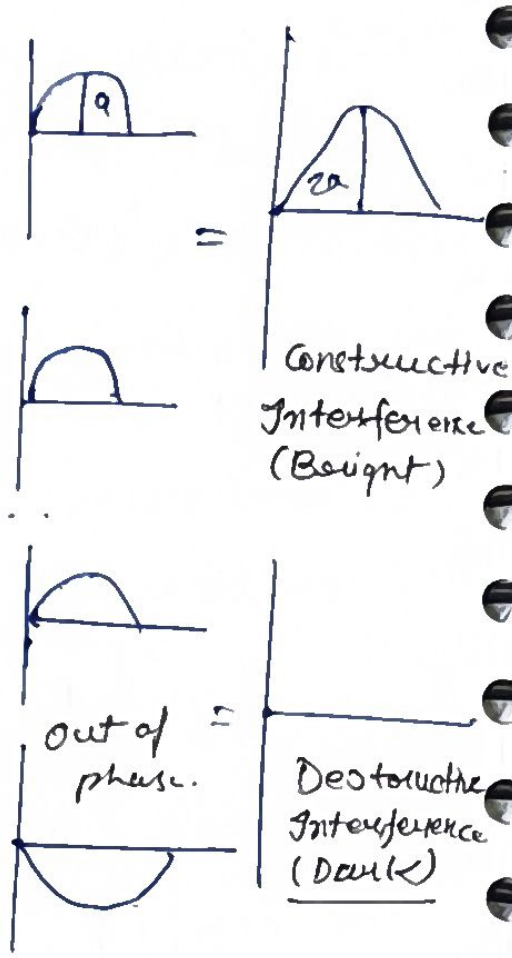
$$\Delta x = n\lambda \quad (n = 0, 1, 2, 3 \ldots)$$

$$\Delta x = 0, \lambda, 2\lambda, 3\lambda, 4\lambda \ldots$$

Destructive Interference

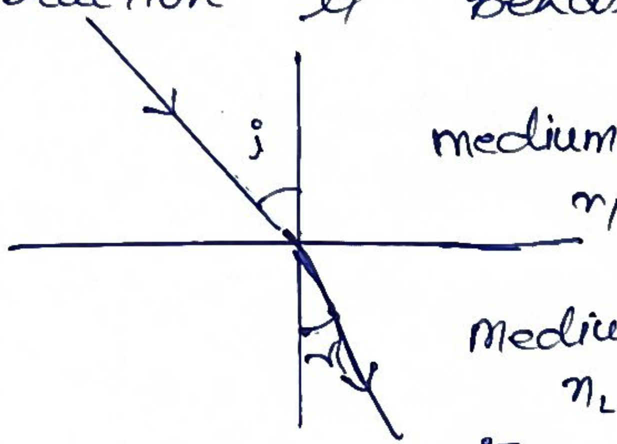
$$\Delta x = (2n-1)\frac{\lambda}{2}, \quad n = 1, 2, 3, 4 \ldots$$

$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \cancel{\frac{4\lambda}{2}}, \frac{5\lambda}{2}, \frac{7\lambda}{2}$$



Refration of light :-

When light goes from rarer to denser medium after refraction it bends towards normal.



$i > r$

i = angle of incidence

r = angle of refraction

~~Snell's~~

Snell's law

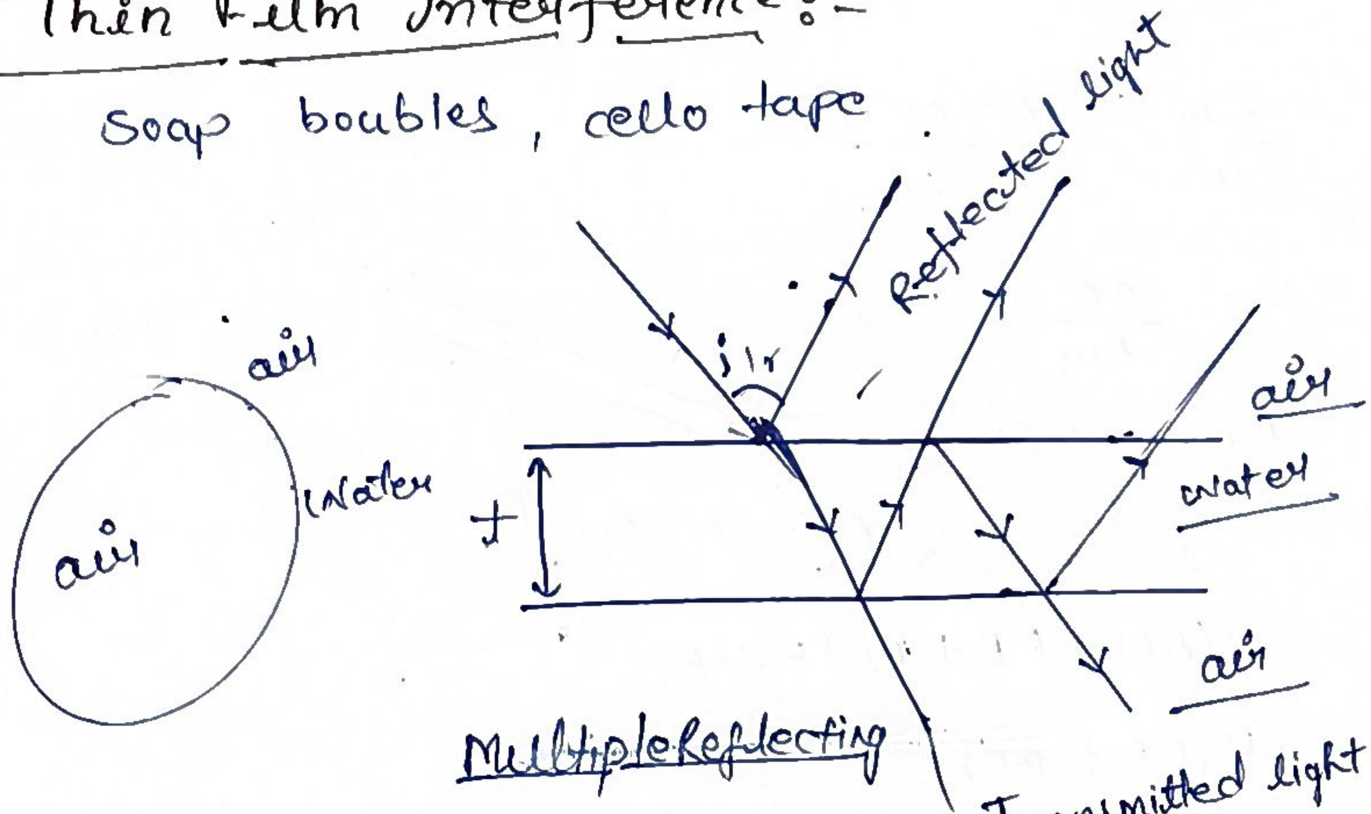
$$n_1 \times \sin i = n_2 \times \sin r$$

$$\mu = \frac{\sin i}{\sin r} = \frac{n_2}{n_1} = n_2$$

μ = refractive index

Thin Film Interference :-

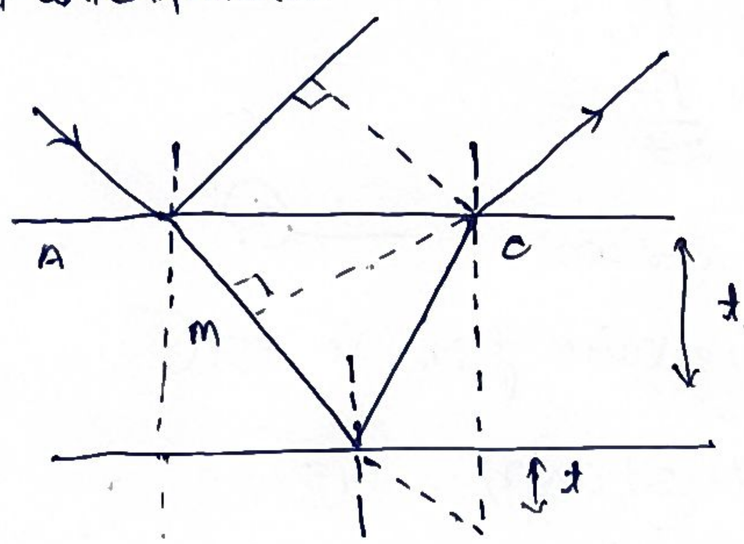
Soap boubles, cello tape



Multiple Reflecting

t = thickness of the film.

Refelected light interference



Optical path difference between rays ① + ②

$$\Delta X = \mu(AB + BC) - AK$$

$$\Delta X = \mu(AM + MB + BC) - AK \quad \text{--- ①}$$

$\Delta AKC =$

$$\sin i = \frac{AK}{AC}$$

ΔAMC : $\sin r = \frac{AM}{AC}$

$$\frac{\sin i}{\sin r} = \frac{AK/AC}{AM/AC}$$

$$\mu = \frac{AK}{AM}$$

$$AK = \mu AM \quad \text{--- ②}$$

putting value of AK in eqn ①

$$\Delta X = \mu(AM + MB + BC) - \mu AM$$

$$\Delta X = \mu(MB + BC)$$

$$\Delta X = \mu(MB + BP) \quad | BC = BP$$

$$\Delta X = \mu(PM) \quad \text{--- ③}$$

ΔPMC $$\cos r = \frac{PM}{PC}$$

$$\cos r = \frac{PM}{2t}$$

$$PM = 2t\cos r \quad \text{--- ④}$$

putting value of PM in eq ③

$$\Delta X = \mu(2t\cos r) \quad \text{--- ⑤}$$

By Stoke's law, since ray ① reflect at denser interface on extra path difference $\frac{\lambda}{2}$ take place

Actual path difference.

$$\Delta x = 2\mu t \cos r - \frac{\lambda}{6} \quad \text{——— (6)}$$

$$2\mu t \cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda - \frac{\lambda}{2}$$

$$\boxed{2\mu t \cos r = n\lambda}$$

$$n = 1, 2, 3, \ldots\ldots$$

Que - Light of wavelength 5893Å is reflected at normal incidence from a soap film of reflective index 1.42. What is the least thickness of the film that will appear (a) bright and (b) dark?

$\lambda = 5893\,\mathring{A}$ (Sodium light).

normal incidence $i = r = 0$

$\mu = 1.42$

$t = ?$

(a) Bright

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2} \quad (n = 0, 1, 2, 3, 4, \ldots\ldots)$$

for least thickness $n = 0$

$$2 \times 1.42 \times t \times \cos 0 = \frac{\lambda}{2}$$

$$2\mu t \cos 0 = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{4\mu}$$

$$t = \frac{5893}{4 \times 1.42}\,\mathring{A}$$

$$t = \cancel{2092.075}\ 1037.5$$

$$\boxed{t = 1037.5}$$

(b) Dark

$$2\mu t \cos r = n\lambda \quad (n = 1, 2, 3 \ldots\ldots)$$

$$n = 1$$

$$2\mu t \cos 0 = \lambda$$

$$t = \frac{\lambda}{2\mu}$$

$$= \frac{5893}{2 \times 1.42}$$

$$\boxed{t = 2075\,\mathring{A}}$$

Que- A thin film of soap solution is illuminated by white light at an angle of incidence, $i = \sin^{-1}\frac{4}{5}$

(2) in reflected light, two dark ~~osctt~~ consecutive fringes are observed overlapping corresponding to wavelengths 6.1×10^{-7} and 6.0×10^{-7} m.

The refractive index for soap solution is $4/3$.

calculate the thickness of the film -

$$i = \sin^{-1}\left(\frac{4}{5}\right) \qquad \mu = 4/3$$

$$\sin i = \frac{4}{5} \qquad t = ?$$

Dark :-

$$2\mu t \cos r = n\lambda \quad (1, 2, 3, 4 \ldots)$$

$$\mu = \frac{\sin i}{\sin r}$$

$$2\mu t \cos r = n\lambda_1 \quad \text{——— (1)}$$

$$2\mu t \cos r = (n+1)\lambda_2 \quad \text{——— (2)}$$

$$\lambda_1 = 6.1 \times 10^{-7} m$$

$$\lambda_2 = 6.0 \times 10^{-7} m$$

$$n\lambda_1 = (n+1)\lambda_2$$

$$n\lambda_1 = n\lambda_2 + \lambda_2$$

$$n(\lambda_1 - \lambda_2) = \lambda_2$$

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

$$n = \frac{6.0 \times 10^{-7}}{0.1 \times 10^{-7}}$$

$$n = 60$$

$$\cancel{2\mu t \cos\theta = n\lambda_1}$$

$$\frac{\sin i}{\sin r} = \mu$$

$$\sin r = \frac{\sin i}{\mu} = \frac{4/5}{4/3}$$

$$\boxed{\sin r = 3/5}$$

$$\cos r = \sqrt{1 - \sin^2 r}$$

$$= \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

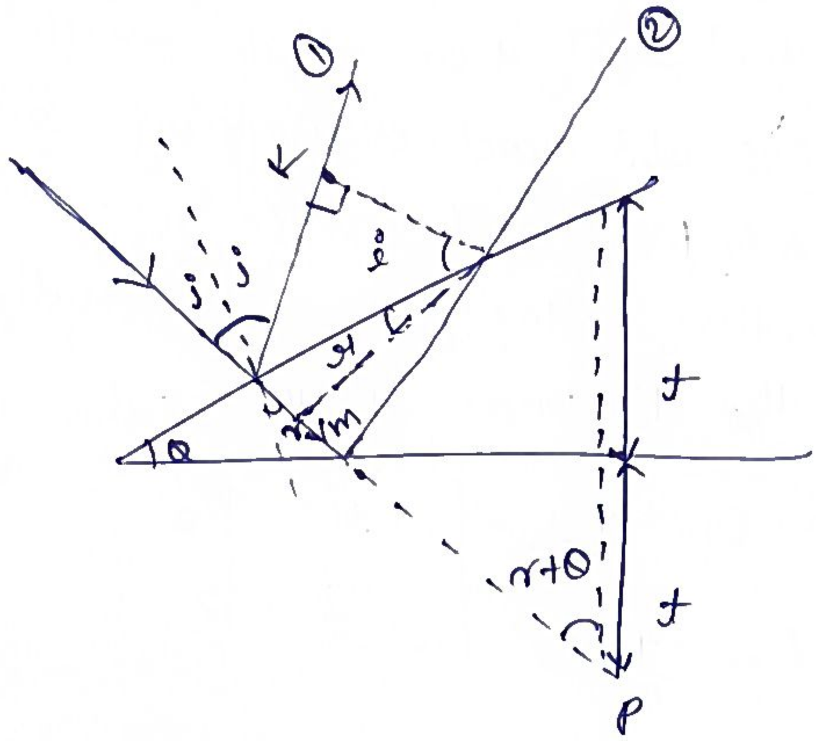
$$\cos r = 4/5$$

eqn (1) $2\mu t \cos r = n\lambda_1$

$$t = \frac{n\lambda_1}{2\mu \cos r} = \frac{60 \times 6.1 \times 10^{-7}}{2 \times \frac{4}{3} \times \frac{4}{5}}$$

$$\boxed{t = 1.7 \times 10^{-5} m}$$

Wedge - Shaped Thin film :- Variable Thickness



let a wedge shaped thin film of wedge angle θ and reflective index μ

① + ②

$$\Delta x = \mu(AB + BC) - AK$$

$$\Delta x = \mu(AM + MD + BC) - AK \quad \text{—— ①}$$

ΔAKC : $\sin i = \frac{AK}{AC}$

ΔAMC $\sin r = \frac{AM}{AC}$

$$\frac{\sin i}{\sin r} = \frac{AK/AC}{AM/AC}$$

$$\mu = \frac{AK}{AM}$$

$$AK = \mu AM \quad \text{—— ②}$$

putting value of AK in eqn ①

$$\Delta x = \mu(AM + MD + BC) - \mu AM$$

$$\Delta x = \mu(MB + BC)$$

$$\Delta x = (MB + BP) \quad | \quad BC = BP$$

$\Delta X = \mu pm$ —— (3)

ΔPML

$$\cos(r+\theta) = \frac{pm}{pc}$$

$$pm = pc\cos(r+\theta) = 2t\cos(r+\theta)$$

~~For bright fri~~

putting value of PM in eqn (3)

$$\Delta x = \mu[2t\cos(r+\theta)]$$

$$\Delta x = 2\mu t\cos(r+\theta) \quad \text{—— (4)}$$

By stoke's law, since ray (1) reflected at dis denser interference, an extra Path difference $\lambda/2$ take place.

∴ Actual Path difference —— (5)

$$\Delta x = 2\mu t\cos(r+\theta) - \lambda/2$$

For bright fringe

$$\Delta x = n\lambda \quad (n = 0, 1, 2, 3, 4, \ldots)$$

$$2\mu t\cos(r+\theta) - \frac{\lambda}{2} = n\lambda$$

$$2\mu t\cos(r+\theta) = n\lambda + \frac{\lambda}{2}$$

$$= \frac{(2n\lambda + \lambda)}{2}$$

$$\boxed{2\mu t\cos(r+\theta) = (2n+1)\frac{\lambda}{2}}$$

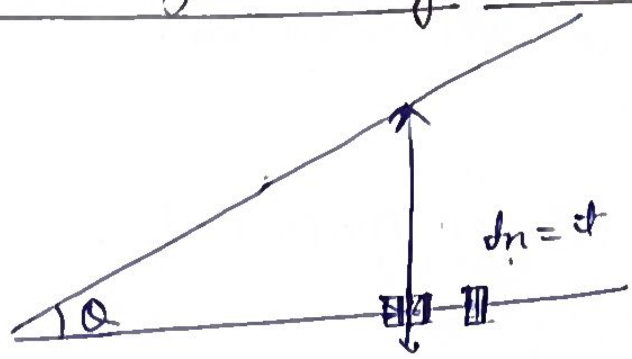
$n = 0, 1, 2, 3$

For dark

$$\Delta x = (2n-1)\frac{\lambda}{2}$$

$$2\mu t\cos(r+\theta) - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

Expression for Fringe Width :-



X_n = Distance of n^{th} dark fringe

$t = t_n$ = thickness of the film

$$\tan\theta = \frac{t_n}{X_n}$$

~~tan tan~~ $t_n = \tan\theta \, X_n$

For n^{th} dark fringe.

$2\mu t \cos(r+\theta) = n\lambda \quad (n = 1, 2, 3, 4.)$.

Putting value of $t = t_n = X_n \tan\theta$

$$2\mu X_n \tan\theta \cos(r+\theta) = n\lambda$$

$$X_n = \frac{n\lambda}{2\mu \tan\theta \cos(r+\theta)} \quad \text{——— (1)}$$

Next fringe $n = n+1$

$$X_{n+1} = \frac{(n+1)\lambda}{2\mu \tan\theta \cos(r+\theta)} \quad \text{——— (2)}$$

Fringe width $\beta = X_{n+1} - X_n$ = Distance between two successive fringes.

$$\beta = \frac{\lambda}{2\mu \tan\theta \cos(r+\theta)}$$

For nor~~malization~~mal incidence

$r = 0$

$$\beta = \frac{\lambda}{2\mu \tan\theta \cos\theta}$$

$$\beta = \frac{\lambda}{2\mu \tan\theta \cos\theta}$$

$$\beta = \frac{\lambda}{2\mu \sin\theta}$$

For small wedge angle

$\theta \to 0 \Rightarrow \sin\theta \simeq \theta$

$$\boxed{\beta = \frac{\lambda}{2\mu\theta}}$$

For air film $\mu = 1$

$$\boxed{\beta = \frac{\lambda}{2\theta}}$$

β depends on

$$\beta = \frac{\lambda}{2\mu\theta}$$

$\beta \propto \lambda$

$\beta \propto \frac{1}{\mu}$ ($\lambda_R = 7000 Å$)

$\beta \propto \frac{1}{\theta}$ ($\lambda_V = 4000 Å$)

Que- Two plane glass surfaces in contact along one edge are separated at the opposite edge by a thin wire. If 20 of interference fringes are observed between these ~~edegs~~ edges in sodium in sodium light (5890 Å) at normal incidence, what is the thickness of the wire?

Que - Two glass plates enclose a wedge shaped air film, touching at one edge and are separated by a wire of 0.05 mm diameter at a distance of 15cm ~~for~~ from the edge. Calculate the fringe width. Monochromatic light of $\lambda = 6000 \mathring{A}$ from a broad source falls normally on the film.

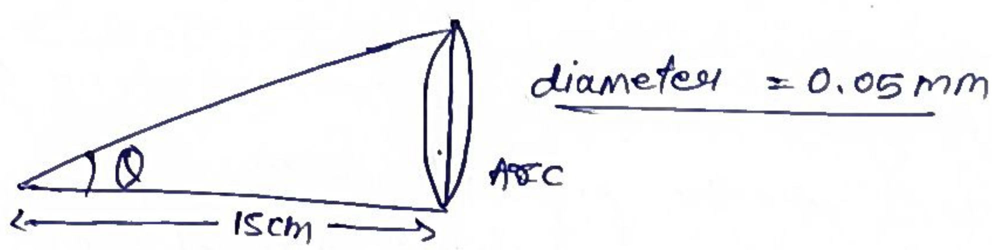
①

Sol^n $\beta = \dfrac{\lambda}{2\mu\theta}$

$\lambda = 6000 \mathring{A} = 6000 \times 10^{-10}$ m

$\beta = ?$

$\mu = 1$ (air film)



$\theta^c = \dfrac{Arc}{Radius} = \dfrac{0.05 \times 10^{-3}}{15 \times 10^{-2}} = \dfrac{1}{30} \times 10^{-3}$

$\beta = \dfrac{\lambda}{2\mu\theta} = \dfrac{6000 \times 10^{-10}}{2 \times 1 \times \frac{1}{3} \times 1\ 10^{-3}}$

$\boxed{\beta = 9 \times 10^{-4} m}$

Que - light of wavelength 6000 Å falls normally on a wedge-shaped film of refractive index 1.4 forming fringes that are 2.00mm a part. Find the angle of wedge in seconds.

$\theta = ?$ (second)

$\lambda = 6000 \mathring{A} = 6000 \times 10^{-10}$ m

$\mu = 1.4$

$\beta = 2 \times 10^{-3}$ m

$$\beta = \frac{\lambda}{2\mu\theta}$$

$$\theta = \frac{\lambda}{2\mu\beta} = \frac{(6000\times10^{-10})}{(2\times1.4\times2\times10^{-3})}$$

$$\theta = 1.071\times10^{-4}$$

$$\pi^c = 180^\circ$$

$$\pi^c = (1800\times60\times60)''$$

$$1^c = \frac{(180\times60\times60)}{\pi}$$

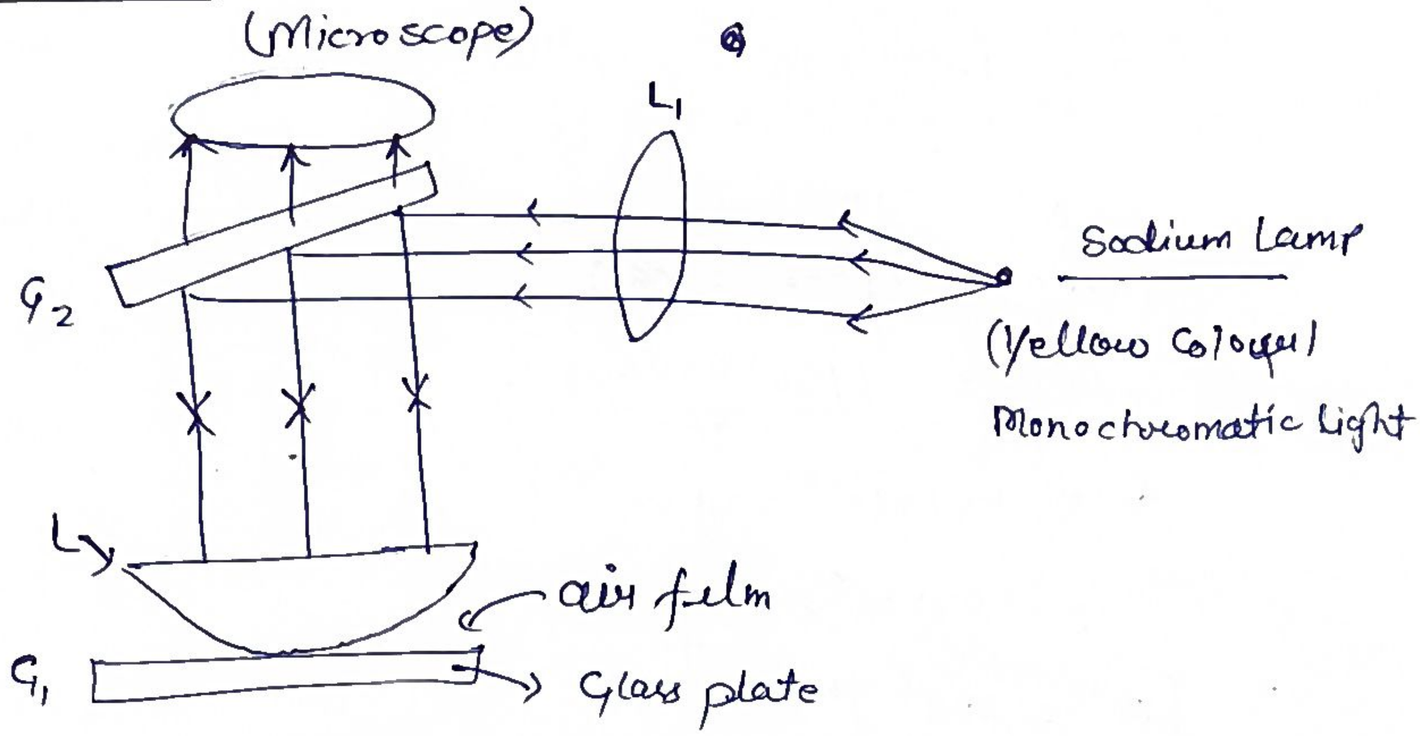
$$\theta = 1.07\times10^{-4}\ rad$$

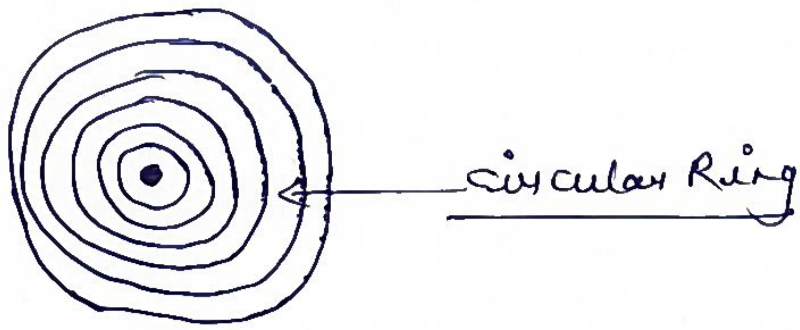
$$= 1.07\times10^{-4}\times\frac{180\times60\times60}{\pi}$$

$$\boxed{\theta = 22''}$$

NEWTON'S RING :-

Experiment





Theory →

For dark fringe

$$2\mu t\cos(r+\theta) = n\lambda \qquad (1)$$

$(1, 2, 3, 4 \ldots\ldots)$

For bright fringe

$$2\mu t\cos(r+\theta) = (2n+1)\frac{\lambda}{2} \qquad (2)$$

$(n = 0, 1, 2, 3 \ldots)$

For normal incidence

$r = 0$

~~for st~~

For small angle wedge angle

$0 \to \theta$

$\cos(r+\theta) = \cos 0 = 1$

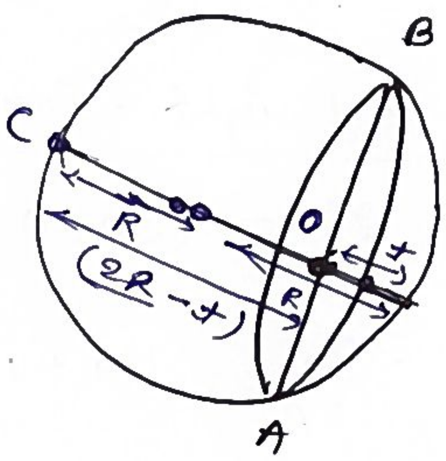
For air film

$\mu = 1$

Dark $= 2t = n\lambda$ ——— (3)

Bright $2t = (2n+1)\frac{\lambda}{2}$ — (4)

By theorem of circle



$OA \cdot OB = OC \cdot OD$

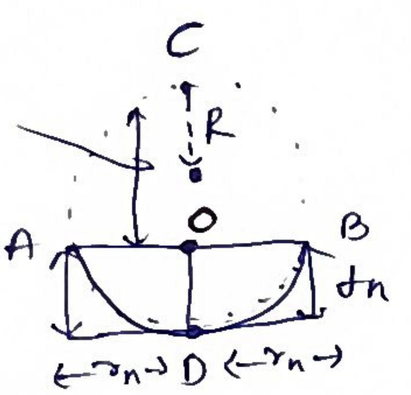
$r_n \cdot r_n = (2R - t_n) \times t_n$

$r_n^2 = (2R - t_n) \times t_n$

As $R >> t_n$

$r_n^2 = 2R \times t_n$

$t_n = \frac{r_n^2}{2R}$ ——— (5)



R = radius of curvature of plano-convex lens

t_n = Thickness of air film

r_n = radius of n^{th} ring

For dark ring

$t = t_n$

$2t_n = n\lambda \quad (n = 1, 2, 3 \ldots\ldots)$

$2\frac{r_n^2}{2R} = n\lambda$

$\frac{r_n^2}{R} = n\lambda$

$r_n^2 = n\lambda R$

$\boxed{r_n = \sqrt{n\lambda R}}$ $n = 1, 2, 3 \ldots$

Diameter $D_n = 2r_n$

$$D_n = 2\sqrt{n\lambda R}$$

$$D_n \propto \sqrt{n} \qquad \underline{n = 1, 2, 3}$$

This show that diameter of dark ring is proportional to square root of natural numbers.

<u>For Bright Ring</u>

$$2t = (2n+1)\frac{\lambda}{2}$$

$$2\frac{r_n^2}{2R} = (2n+1)\frac{\lambda}{2}$$

$$\frac{r_n^2}{R} = (2n+1)\frac{\lambda}{2}$$

$$r_n^2 = (2n+1)\frac{\lambda R}{2}$$

$$\boxed{r_n^2 = \sqrt{(2n+1)\frac{\lambda R}{2}}}$$

Diameter

$$D_n = 2r_n = 2\sqrt{(2n+1)\frac{\lambda R}{2}}$$

$$n = 0, 1, 2, 3 \ldots\ldots$$

$$D_n \propto \sqrt{(2n+1)}$$

$$D_n \propto \sqrt{1, 3, 5, 7 \ldots\ldots}$$

This shows that diameter of bright ring is proportional to square root of natural odd numbers.

Determination of Wave length of Sodium light :-

Radius of n^{th} dark ring.

$$r_n = \sqrt{n\lambda R}$$

Diameter $D_n = 2r_n = 2\sqrt{n\lambda R}$

$$D_n^2 = 4n\lambda R \quad \text{——— (1)}$$

$$n = n+p$$

$$(D_{n+p})^2 = 4(n+p)\lambda R \quad \text{— (2)}$$

$$(D_{n+p})^2 - D_n^2 = 4(n+p)\lambda R - 4n\lambda R$$

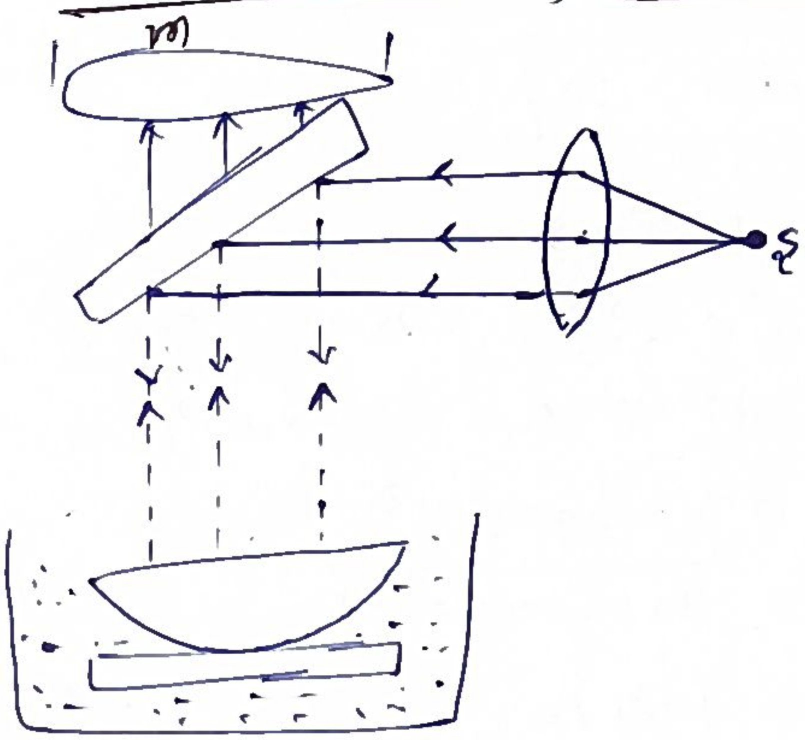
$$(D_{n+p})^2 - D_n^2 = 4P\lambda R$$

$$\lambda = \frac{(D_{n+P})^2 - D_n^2}{4PR}$$

$n = 10 \quad P = 8$

$$\lambda = \frac{D_{18}^2 - D_{10}^2}{4 \times 8 \times R}$$

Determination of Refractive Index of Transparent liquid



Radius of n^{th} dark ring.

$$r_n = \sqrt{\frac{n\lambda R}{\mu}}$$

Diameter

$$D_n = 2r_n = 2\sqrt{\frac{n\lambda R}{\mu}}$$

$$D_n^2 = \frac{4n\lambda R}{\mu} \quad \text{——— (1)}$$

$$n = n+p$$

$$(D_{n+p})^2 = \frac{4(n+p)\lambda R}{\mu} \quad \text{---} ②$$

$$(D_{n+p})^2 - (D_n)^2 = \frac{4p\lambda R}{\mu} \quad \text{---} ③$$

for liquid (l)

$$(D_{n+p})_l^2 - (D_n)^2 = \frac{4p\lambda R}{\mu} \quad \text{---} ④$$

for air ($\mu = 1$)

$$(D_{n+p})^2 - (D_n)^2 = 4p\lambda R \quad \text{---} ⑤$$

putting value of eq^n ⑤ in eq^n ④

$$(D_{n+p})_l^2 - (D_n)_l^2 = \frac{(D_{n+p})_a^2 - (D_n)_a^2}{\mu}$$

$$\boxed{\mu = \frac{(D_{n+p})_a^2 - (D_n)_a^2}{(D_{n+p})_l^2 - (D_n)_l^2}}$$

Que - Newton's Ring are observed in reflected light of wave length 6000Å. The diameter of 10th dark ring is 0.5cm. Find the radius of curvature of the lens and the thickness of the air film.

Sol^n

$R = ?$

$t = ?$

$\lambda = 6000Å$

$D_{10} = D_n = 0.5cm$

Radius of nth dark ring

$r_n = \sqrt{n\lambda R}$ ($n = 1, 1, 3, 4, ...$)

$n = 10$

$r_n^2 = n\lambda R$

$$R = \frac{r_n^2}{n\lambda}$$

$D_{10} = 0.5\ cm$

$r_{10} = \frac{0.5}{2} = 0.25cm = 0.25 \times 10^{-2} m$

$\lambda = 6000Å = 6000 \times 10^{-10} m$

$$R = \frac{(0.25 \times 10^{-2})^2}{(10 \times 6000 \times 10^{-10})}$$

$R = 1.04\ m$

$$t_n = \frac{r_n^2}{2R}$$

$$= \frac{(0.25 \times 10^{-2})^2}{(2 \times 1.04)}$$

$$\boxed{t_n = 3.00 \times 10^{-6}}$$

(3)

Que - Newton's ring are observed by keeping a spherical surface of 100cm radius on a plane glass plate. If the diameter of 15th bright ring 0.590cm and the diameter of 5th ring is 0.336cm. What is the wave length of light used.

$R = 100 cm.$ - $R = 100 \times 10^{-2} m$

$D_{15} = D_{n+p}$ 0.0590cm

$D_5 = D_n = 0.336 cm.$

$p = 10$

$$\lambda = \frac{(D_{n+p})^2 - (D_n)^2}{4pR}$$

$$= \frac{(0.590)^2 - (0.339)^2}{(4 \times 10 \times 100)}$$

$$\boxed{\lambda = 5.88 \times 10^{-5} \text{ cm}}$$

$$\boxed{\lambda = 5.880 \times 10^{-7}}$$

$$\boxed{\lambda = 5880 Å}$$

Que - In newton's ring experiment the diameter of 4th and 12th dark ring are 0.400 cm and 0.700cm respectively. Deduce the diameter of of 20th dark ring.

$D_4 = 0.400 cm$

$D_{12} = 0.700 cm$

$D_{20} = ?$

Radius of nth dark ring

$r_n = \sqrt{n \lambda R}$

Diameter $(D_n) = 2\sqrt{n \lambda R}$

$D_n^2 = 4 n \lambda R$

$D_4^2 = (4 \times 4) \lambda R = 16 \lambda R$ — (1)

$D_{12}^2 = \cancel{124 \times 1}(12 \times 4) \lambda R = 48 \lambda R$ — (2)

$D_{20}^2 = (20 \times 4) \lambda R = 80 \lambda R$ — (3)

$D_{12}^2 - D_4^2 = D_{20}^2 - D_{12}^2$

$D_{20}^2 = 2D_{12}^2 - D_4^2$

$= 2 \times (0.700)^2 - (0.400)^2$

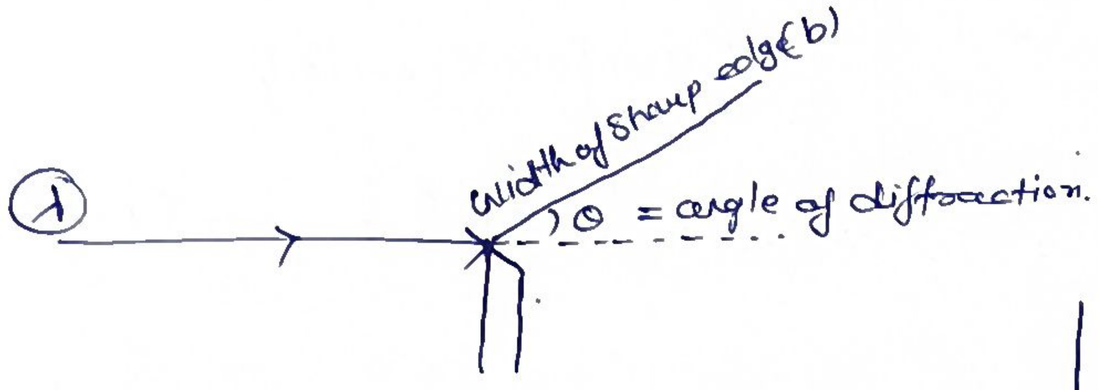
$D_{20}^2 = 0.82$

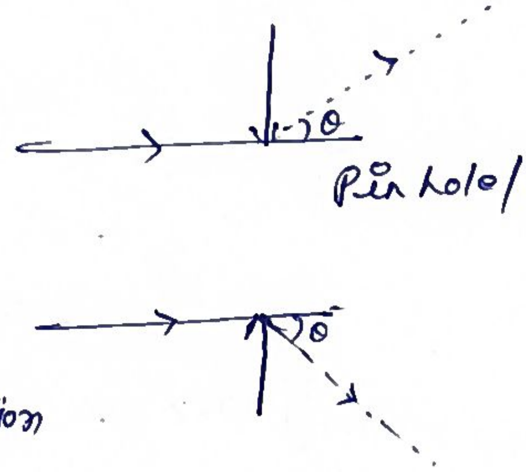
$D_{20} = \sqrt{.82} = .905$

$\boxed{D_{20} = 0.905 cm}$

Diffraction of light :-

(1)

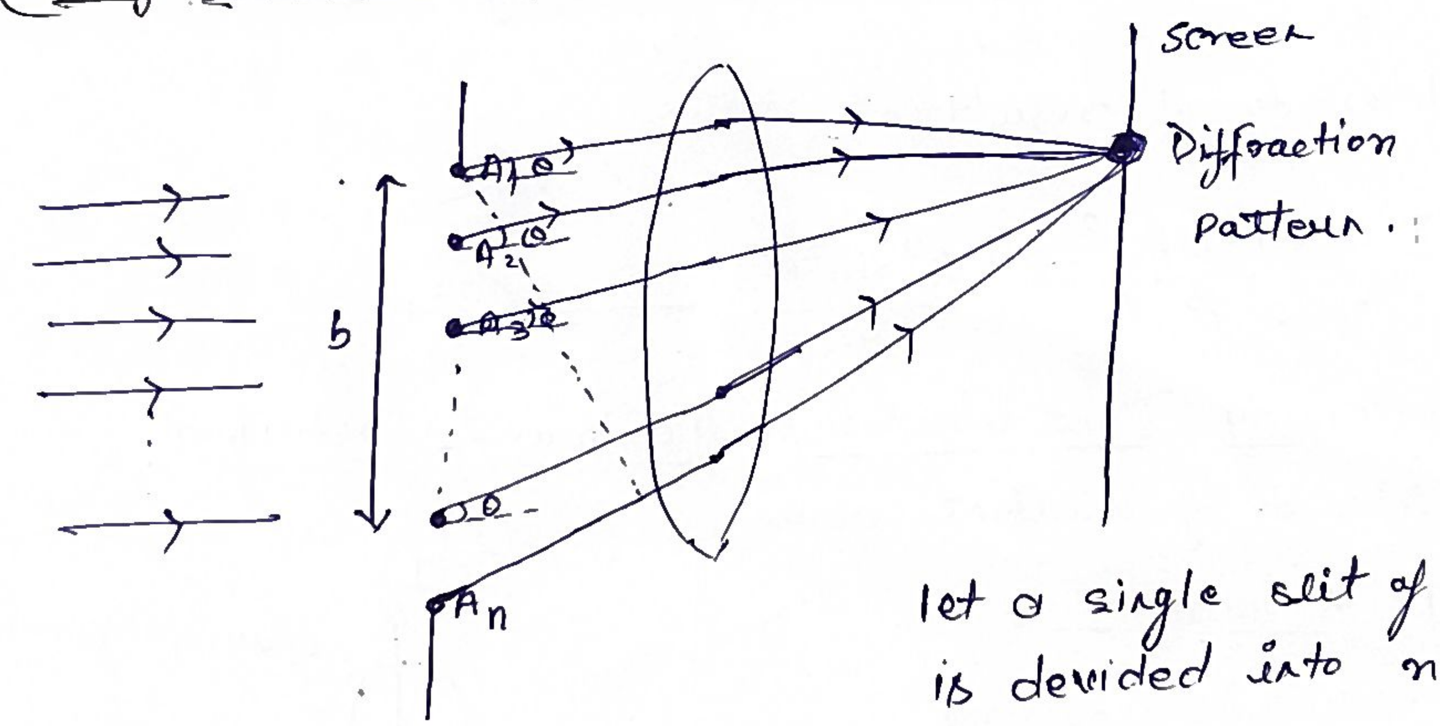




Bending of light at ~~an~~ ~~step~~ sharp edge of opaque obsolete is called diffraction of light.

Spreading out of light when passing through pin hole is called diffraction of light.

Single split fraunhofer Differaction :-



let a single slit of width b, is devided into n-point sources.

Wave ~~stating~~ starting from each point sources reach the screen at ~~p~~ point P.

R.

Resultant wave

$$E = \cancel{a\sin}\ a\cos\omega t + a\cos(\omega t - \phi)$$
$$+ \cdots\cdots + a\cos\{\omega t - (n-1)\phi\}$$

$$E = \frac{a\sin\frac{n\phi}{2}\cos\{\omega t - \frac{1}{2}(n-1)\phi\}}{\sin\frac{\phi}{2}} \longleftarrow (1)$$

let $\frac{n\phi}{2} = \beta = \pi\frac{b\sin\theta}{\lambda}$

$$\frac{\phi}{2} = \frac{\beta}{n}$$

if. $n \to \infty$, $\frac{\phi}{2} \to 0$ | $\sin\frac{\phi}{2} = \frac{\phi}{2}$

$$\therefore E = \frac{a\sin\beta}{\frac{\beta}{n}}\cos(\omega t - \beta)$$

$$E = \frac{A\sin\beta}{\beta}\cos(\omega t - \beta) \quad \text{--- (2)}$$

Where $A = na$ = amplitude of n waves.

Intensity of resultant wave

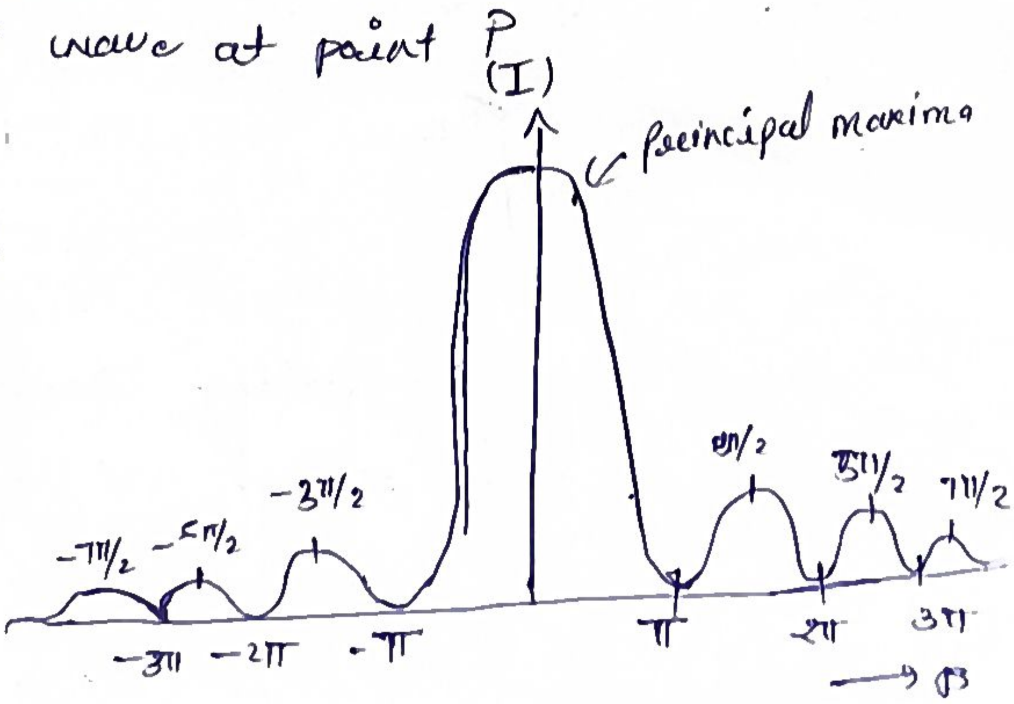
$$I = \left(\frac{A\sin\beta}{\beta}\right)^2 = \frac{A^2\sin^2\beta}{\beta^2} \longleftarrow (3)$$

Intensity Distribution :- Maxima & Minima

Intensity of resultant wave at point P

$$I = \frac{A^2\sin^2\beta}{\beta^2} \quad \text{--- (4)}$$

where $\beta = \frac{\pi b\sin\phi}{\lambda}$



For Minimum Intensity Minima (zero intensity)

$$I = \frac{A^2 \sin^2\beta}{\beta^2}$$

$$I_{min} = 0$$

$$0 = \frac{A^2 \sin^2\beta}{\beta^2}$$

$$\frac{\sin\beta}{\beta} = 0 \quad (\beta \neq 0)$$

$$\sin\beta = \sin m\pi \quad (\text{where } m = \pm 1, \pm 2, \pm 3, \ldots)$$

$$\beta = m\pi$$

$$\frac{\pi b \sin\theta}{\lambda} = m\pi$$

$$\boxed{b\sin\theta = m\lambda}$$

$m = 1$, first minima

$m = 2$, second minima

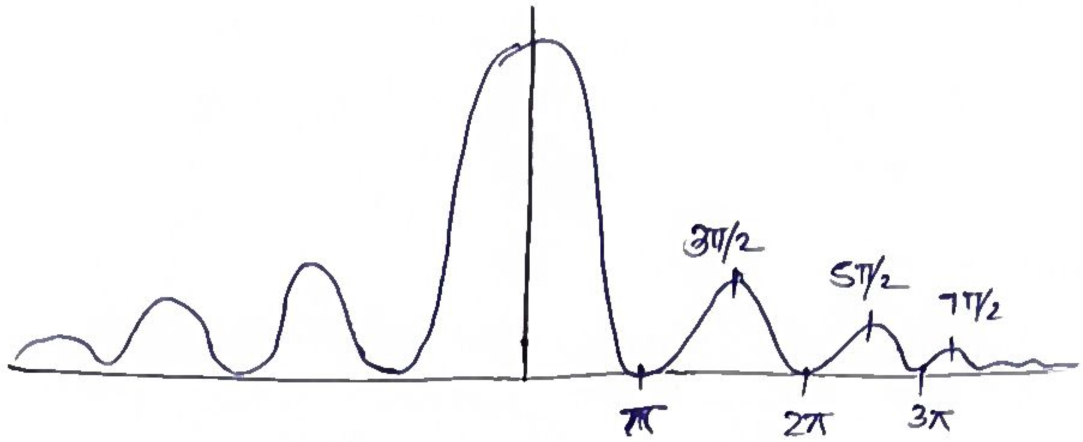
for ~~mi~~ Principal maxima :- Central maxima

if $\beta = 0$, $\frac{\sin\beta}{\beta} = \frac{0}{0}$ (undefined)

$$\lim_{\beta \to 0} \frac{\sin\beta}{\beta} = 1$$

$$I = \frac{A^2 \sin^2\beta}{\beta^2}$$

$$I = A^2 \text{ (maximum)}$$



for secondary maxima

$$I = \frac{A^2 \sin^2\beta}{\beta^2}$$

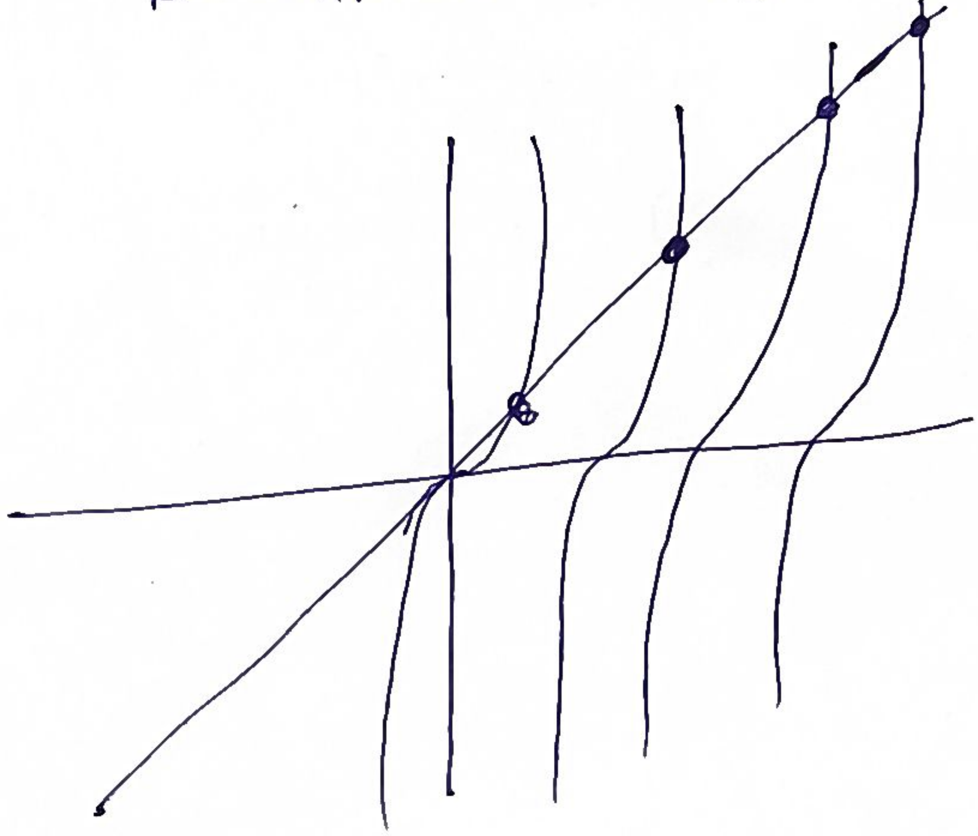
$$\frac{dI}{d\beta} = A^2 \frac{d}{d\beta}\left(\frac{\sin^2\beta}{\beta^2}\right)$$

$$0 = \frac{A^2\left[\beta^2 \frac{d}{d\beta}(\sin^2\beta) - \sin^2\beta \frac{d}{d\beta}(\beta^2)\right]}{(\beta^2)^2}$$

$$0 = \frac{A^2}{\beta^4}\left[\beta^2(2\sin\beta\cos\beta) - \sin^2\beta(2\beta)\right]$$

$$0 = 2\beta\sin\beta\cos\beta\,[\beta - \tan\beta]$$

$$\beta - \tan\beta = 0 \quad \text{------} \quad (5)$$



$\beta = 0$, $I_0 = A^2$ (Principal Maximum)

First Secondary maxima : $\beta = 3\pi/2$

$$I_1 = \frac{A^2 \sin^2 3\pi/2}{(3\pi/2)^2} = A^2 \frac{4}{9\pi^2}$$

Second Secondary Maxima

$$\beta = 5\pi/2$$

$$I_2 = \frac{A^2 \sin^2 \frac{5\pi}{2}}{(5\pi/2)^2} = A^2 \frac{4}{25\pi^2}$$

Third Secondary maxima

$$\beta = \frac{7\pi}{2}$$

$$I_3 = \frac{A^2 \sin^2 \frac{7\pi}{2}}{\left(\frac{7\pi}{2}\right)^2} = \frac{A^2\, 4}{49\pi^2}$$

Relative intensities of secondary maxima

$$I_0 : I_1 : I_2 : I_3 : I_4 \ldots\ldots = A^2 : A^2\frac{4}{9\pi^2} : A^2\frac{4}{25\pi^2} : A^2\frac{4}{49\pi^2}$$

$$I_0 : I_1 : I_2 : I_3 : I_4 \ldots\ldots = 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} : \frac{4}{81\pi^2} : \ldots\ldots$$

(show that intensity of first secondary maxima is only 5% of principal maxima)

$$I_0 = A^2$$

$$I_1 = A^2 \frac{4}{9\pi^2}$$

$$\frac{I_1}{I_0} \times 100 = \frac{4}{9\pi^2} \times 100$$

$$\frac{I_1}{I_0} \times 100 = 4.5\%$$

$$= 5\%$$

This shows that intensity of first secondary maxima is only 5% of principal maxima.

Que-01:- light of wavelength 5500 Å falls normally on a slit of width 2.2×10^{-4} cm. Calculate the angular position of the first two minima on either side of a central maxima.

$\lambda = 5500$ Å $\rightarrow 5500 \times 10^{-10}$ m

$b = 2.2 \times 10^{-4} \rightarrow 2.2 \times 10^{-6}$ m

$b \sin\theta = m\lambda \quad (m = 1, 2, 3 \ldots)$

$m = 1$, first minima

$b \sin\theta = \lambda$

$\sin\theta = \frac{\lambda}{b}$

$\theta = \sin^{-1}\left(\frac{\lambda}{b}\right)$

$= \sin^{-1}\left(\frac{5500 \times 10^{-10}}{2.2 \times 10^{-6}}\right)$

$\underline{\theta = 14.47}$

$\underline{m = 2}$

$b \sin\theta = 2\lambda$

$\sin\theta = \frac{2\lambda}{b}$

$\theta = \sin^{-1}\left(\frac{2\lambda}{b}\right)$

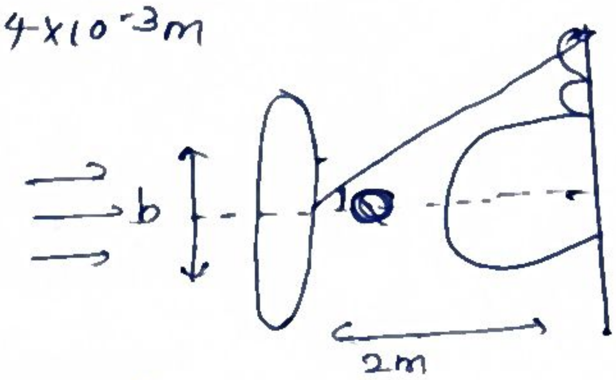
$= \sin^{-1}\left(\frac{2 \times 5500 \times 10^{-10}}{2.2 \times 10^{-6}}\right)$

$\underline{\theta = 30}$

Que-02 → A single slit of width 0.14 mm is illuminated normally by monochromatic light and diffraction bands are observed on a screen 2m way. If the centre of second dark band is 1.6cm from the central bright band, deduce the wavelength of light.

(4)

$b = 0.14\,mm = 0.14 \times 10^{-3}\,m$



$b \sin\theta = m\lambda$

second dark band (minima)

$m = 2$

$b\sin\theta = 2\lambda$

$\sin\theta = \frac{2\lambda}{b}$

$\lambda = \frac{b\sin\theta}{2}$

$= \frac{b\theta}{2}$

θ is small

$\theta = \frac{Arc}{Radius} = \frac{1.6 \times 10^{-2}}{2}$ rad.

$\theta = 0.8 \times 10^{-2}$ rad

$\lambda = \frac{b\theta}{2}$

$= \frac{0.14 \times 10^{-3} \times 0.8 \times 10^{-2}}{2}$

$\lambda = 0.56 \times 10^{-6} \Rightarrow 5.6 \times 10^{-7}\,m$

$\boxed{\lambda = 5600\,Å}$

Que-03 → Calculate the angle at which the first dark band and the next bright band are formed in the ~~Frau~~ Fraunhofer's diffraction pattern of the slit 0.33mm wide. The wavelength of light used is 5890 Å.

$b = 0.33\,mm = 0.33 \times 10^{-3}\,m$

$\lambda = 5890\,Å = 5890 \times 10^{-10}\,m$

$b\sin\theta = m\lambda$

$m = 1$

$b\sin\theta = \lambda$

$\sin\theta = \frac{\lambda}{b}$

$\theta = \sin^{-1}\left(\frac{\lambda}{b}\right)$

$\theta = 0.102°$

Next bright ban

$\beta = 3\pi/2$

$\frac{\pi b\sin\theta}{\lambda} = 3\pi/2$

$b\sin\theta = \cancel{3\pi/2}\ \frac{3\lambda}{2}$

$\sin\theta = \frac{3\lambda}{2b}$

$$\theta = \sin^{-1}\left(\frac{3\lambda}{2d}\right)$$

$$\theta = \sin^{-1}\left(\frac{(3 \times 5890 \times 10^{-10})}{(2 \times 0.33 \times 10^{-3})}\right)$$

$$\boxed{\theta = 0.152^\circ}$$

Diffraction Grating : Plane Transmission Grating

~~It is~~

N - slit Diffraction

It is combination of large no. of equidistant ~~dist~~ slits.

← Transparent (e or a)
← opaqe (d or b)

$e+d = a+b$ = Distance b/w two successive slits)

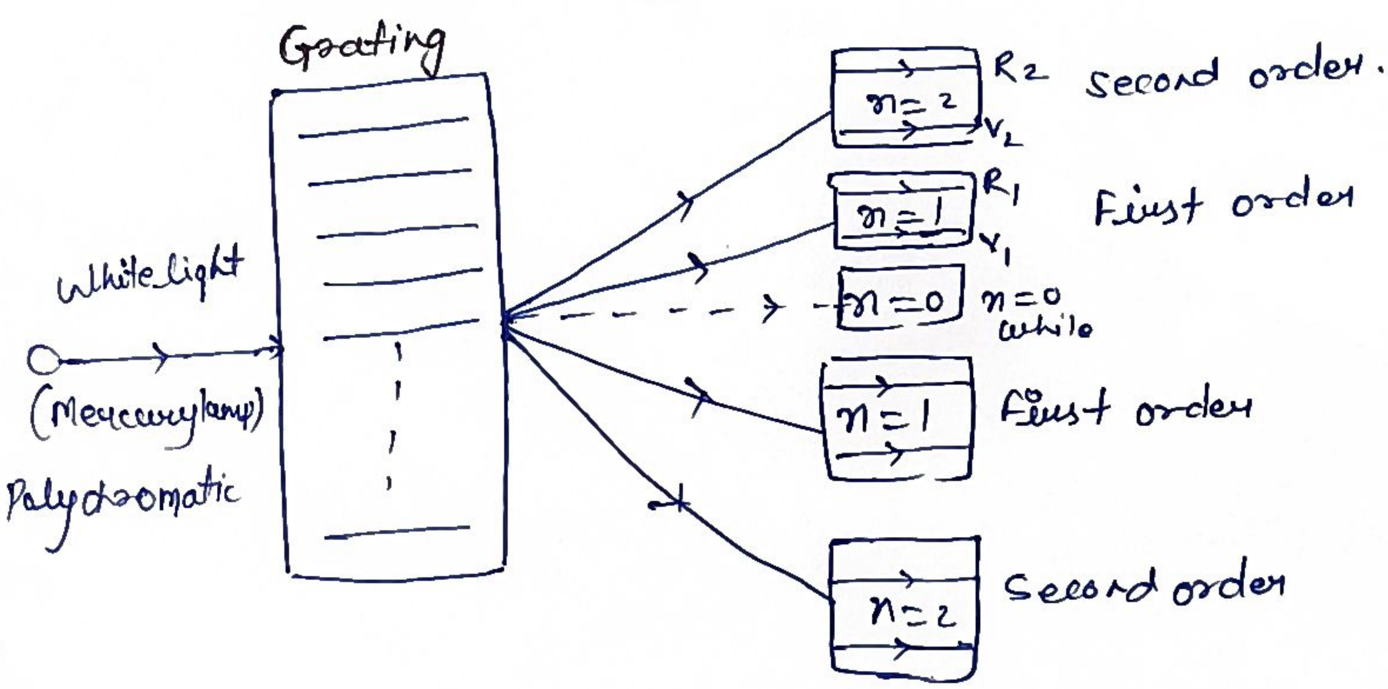
= Grating element

Grating equation : Condition of N- slits interferance maxima.

$(e+d)\sin\theta = n\lambda$ $(n = 0, 1, 2, 3 \ldots)$

$n = 0$ (zero order)
$n = 1$ (First order)
$n = 2$ (second order).

Grating spectrum :- Theorey of Grating.



$(e+d)\sin\theta = n\lambda$

$n=0$, zero-order

$(e+d)\sin\theta = 0$

$\sin\theta = 0$

$$\boxed{\theta = 0}$$

θ does not depend on λ. Therefore all colon's are spotted at a single point. This explain zero-order is white

---

θ depends on λ. Different wave length have different θ. Therefore, colours are s

$n=1$, first order

$(e+d)\sin\theta =$

Dispersive Power of Grating :-

Dispersive power (D.P) = $\frac{d\theta}{d\lambda}$

It is ratio of difference of angles to difference of wavelength.

Grating Eqn →

$$(e+d)\sin\theta = n\lambda \quad \text{——— (1)}$$

$$\frac{d}{d\lambda}\left[(e+d)\sin\theta\right] = \frac{d}{d\lambda}(n\lambda)$$

$$(e+d)\frac{d}{d\lambda}\sin\theta = n$$

$$(e+d)\cos\theta\frac{d\theta}{d\lambda} = n$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta} \quad \text{——— (2)}$$

eqn (1)

$$(e+d)\sin\theta = n\lambda$$

$$\sin\theta = \frac{n\lambda}{e+d}$$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$\cos\theta = \sqrt{1-\left(\frac{n\lambda}{e+d}\right)^2} \quad \text{——— (3)}$$

putting value of $\cos\theta$ in eqn (2)

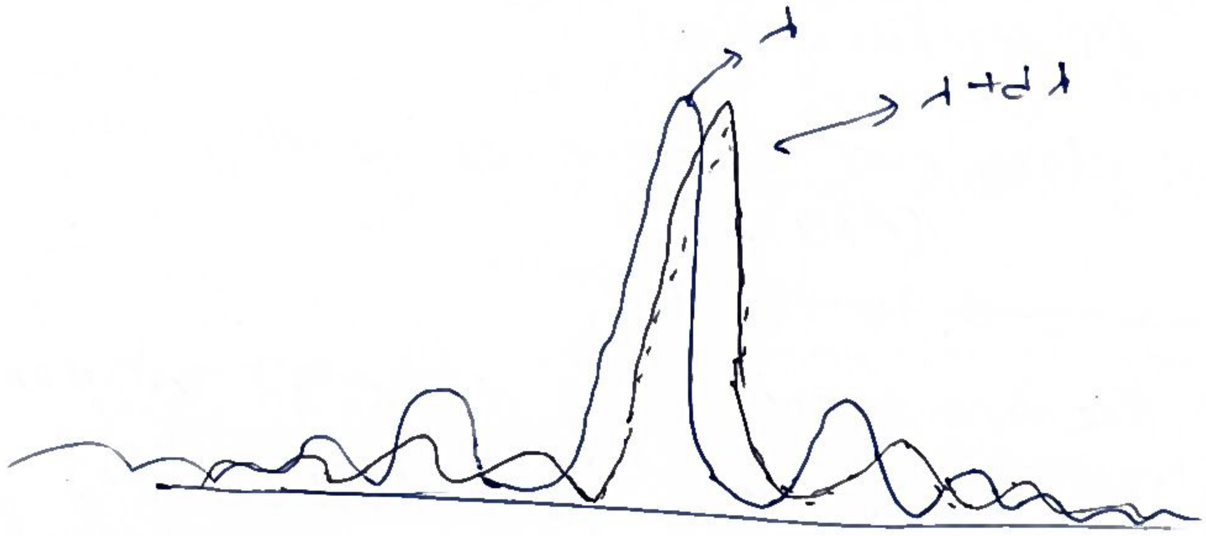
$$\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\left[\sqrt{1-\left(\frac{n\lambda}{e+d}\right)^2}\right]}$$

$$\frac{d\theta}{d\lambda} = \frac{1}{\frac{e+d}{n}\sqrt{1-\frac{n^2\lambda^2}{(e+d)^2}}}$$

$$= \frac{1}{\sqrt{\frac{(e+d)^2}{n^2}\left[1 - \frac{n^2\lambda^2}{(e+d)^2}\right]}}$$

$$\boxed{DP = \frac{1}{\sqrt{\frac{(e+d)^2}{n^2} - \lambda^2}}}$$

Rayleigh Criterian for Resolving Power



To nearby wavelengths λ & $(\lambda + d\lambda)$ are said to be just resolved if diffraction maxima of one coincideas with differaction of maxima of other and vice versa.

Resolving Power of Grating :-

Resolving Power (RP) $= \frac{\lambda}{d\lambda}$

It is ratio of power wavelength to wavelength difference ($d\lambda$).

Two nearby wavelength ~~λ & $\lambda + d\lambda$~~ λ and $\lambda + d\lambda$ are said to be just resolved by grating

if $(e+d)\sin\theta = n(\lambda + d\lambda)$ ——— (1)

$(e+d)\sin\theta = n\lambda + \frac{\lambda}{N}$ ——— (2)

N = total no. of lines on the grating surface.

$n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$

$\cancel{n\lambda} + nd\lambda = \cancel{n\lambda} + \frac{\lambda}{N}$

$nd\lambda = \frac{\lambda}{N}$

$$\boxed{\frac{\lambda}{d\lambda} = nN = RP}$$

$RP \propto n$ (order)

$RP \propto N$ (No of lines)

Note → R.P. does not depend on grating element $(e+d)$.

Que-01 Find the minimum no of lines in a plane transmission grating required to just resolve the sodium doublet 5890 Å and 5896 Å in second order.

$$\frac{\lambda}{d\lambda} = nN$$

$N = ?$

$n = 2$

$\lambda = 5890$

$d\lambda = 5896 - 5890 = 6 Å$

$$= \frac{5890}{6} = 2 \times N$$

$$N = \frac{5890}{12} \quad 490.83$$

$$\boxed{N = 490.83}$$

Ans

Que-02 Can D_1 and D_2 lines of Na light be resolved $\lambda D_1 = 5890$ Å, $\lambda D_2 = 5896$ Å in second order. Number of lines in grating of 2.0 cm wide = 4500.

$\lambda = 5890$ Å

$d\lambda = 6$ Å

$n = 2$

$N = 491$

Yes it is resolved

Que-03 :- A diffraction grating is just able to resolve two lines of wave length 5140.34 Å and 5140.85 Å in the first order. / Will it resolve the lines 8037.2 Å and 8037.5 Å in second order.?

5

Soln

$n = 1$

$\lambda = 5140.34 Å$

$d\lambda = 0.51$

$\frac{\lambda}{d\lambda} = nN$

$N = \frac{\lambda}{d\lambda} = \frac{5140.34}{0.51} =$

$\boxed{N = 10079}$

Given

Required

$n = 2$

$\lambda = 8037.2 Å$

$d\lambda = 0.3 Å$

$\lambda = 8037.2 Å$

$d\lambda = 0.3 Å$

$\frac{\lambda}{d\lambda} = nN$

$N = \frac{8037.2}{2 \times 0.3} = 13395$

No, it can't resolve.

Que-04 A plane transmission grating has 15000 lines per inch. Find the resolving power of the grating and smallest wave length difference that can be resolved with a light of wave-length 6000 Å in the second order.

$N = 15000$

$RP = ?$

$d\lambda = ?$

$\lambda = 6000 Å$

$n = 2$

$RP = nN$

$= 2 \times 15000$

$RP = 30000$

$\frac{\lambda}{d\lambda} = nN$

$d\lambda = \frac{\lambda}{nN} = \frac{6000}{30000}$

$\boxed{d\lambda = 0.2 Å}$

Que-05 A diffraction grating used at normal incidence gives a yellow line ($\lambda = 6000 Å$) in certain spectral order superimposed on a blue line ($\lambda = 4800 Å$) of must higher order. If the angle of diffraction is $\sin^{-1}\left(\frac{3}{4}\right)$ calculate the grating element.

$(e+d) = ?$

$(e+d)\sin\theta = n\lambda$

$(e+d)\sin\theta = n\lambda_y$ — ①

$(e+d)\sin\theta = (n+1)\lambda_b$ — ②

$\lambda_y = 6000 Å$

$\lambda_b = 4800 Å$

$\theta = \sin^{-1}\left(\frac{3}{4}\right)$

$\sin\theta = \frac{3}{4}$

$n\lambda_y = (n+1)\lambda_b$

$n \times 6000 = (n+1)4800$

$n \times 5 = (n+1) \times 4$

$5n = 4n + 4$

$\boxed{n = 4}$

$(e+d)\sin\theta = n\lambda_y$

$(e+d) \times \frac{3}{4} = 4 \times 6000$

$\boxed{(e+d) = 32000 Å}$

$\boxed{e+d = 3.2 \times 10^{-6} M}$

Missing Order Spectrum:- Absent spectra

When interference maxima of N-slits coincides with the diffraction minima of single slits and such orders are missing are called missing order spectrum.

$(e+d)\sin\theta = n\lambda$ (N-Slits interference maxima)

~~e+d~~ $e\sin\theta = m\lambda$ (~~N~~. single slit minima)

$$\frac{e+d}{e} = \frac{n}{m}$$

if transparencies equals to opacities

$$\frac{2e}{e} = \frac{n}{m}$$

$$2 = \frac{n}{m}$$

$$\boxed{n = 2m}$$

$m = 1, 2, 3, 4 \ldots$

$n = 2, 4, 6, 8$ o ~~observe~~ orders are missing

This shows that evens ~~o~~ orders are missing.

opacities = Two lines of Transparencies

$d = 2e$

$$\frac{e+2e}{e} = \frac{n}{m}$$

$$\frac{3e}{e} = \frac{n}{m}$$

$$\boxed{n = 3m}$$

$m = 1, 2, 3, -$

$n = 3, 6, 9 \ldots$

orders are missing