UNIT-01

Quantum Mechnics.

Inadequancy (failurity) of clasical mechanism:

- Stability of atom
- Black body radiation
- photo electric effect
- Compton effect

Wave particle Duality

The properity of light dual nature is called wave particle duality.

De - Broglie Hypothesis:

If the light being wave can show particle nature. similarly a matter particle should also be have like wave.

Matter wave:

Every moving matter particle (eg. e⁻. p, n etc) is associated with a wave are called matter wave.

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

Proof. for particle nature of light

$$E = h\nu$$

$$E = \frac{hc}{\lambda} \quad \text{—(i)}$$

According to Einstein's

$$E = mc^2 \quad \text{—(2)}$$

from eq (I) & (II) we get

$$\frac{hc}{\lambda} = mc^2$$

$$\left[\lambda = \frac{h}{mc}\right]$$ for massless particle

or

$$\left[\lambda = \frac{h}{mv}\right]$$ for massness particle

Wave length in terms of KE :

$$K = \frac{1}{2}mv^2$$

$$2K = mv^2$$

$$2mK = m^2v^2 \qquad [\because p = mv]$$

$$2mK = p^2$$

$$p = \sqrt{2mK}$$

$$\therefore \lambda = \frac{h}{p} \qquad \left[\lambda = \frac{h}{\sqrt{2mK}}\right]$$

wave length in terms of Voltage.

$$\left[K = qV\right]$$

$$\frac{1}{2}mv^2 = qV \qquad \left[\lambda = \frac{h}{\sqrt{2mqV}}\right]$$

$$m^2v^2 = 2mqV$$

$$p^2 = 2mqV$$

$$p = \sqrt{2mqV}$$

for e^-

$$\lambda = \frac{6.623\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}V}}$$

$$\boxed{\lambda = \frac{12.27}{\sqrt{V}}\ Å}$$

wave length in terms of temperature..

for gases

$$K = \frac{3}{2}KT$$

$$\therefore \lambda = \frac{h}{\sqrt{2mK}}$$

$$= \frac{h}{\sqrt{2\times\frac{3}{2}mKT}}$$

$$\boxed{\lambda = \frac{h}{\sqrt{3mKT}}}$$

here

K = Boltzmann Constant $= 1.38\times10^{-23}$ J/K

$h = 6.623\times10^{-34}$ J.s

mass of $e^- = 9.1\times10^{-31}$ kg

mass of proton & netron $= 1.67\times10^{-27}$ kg

mass of α = 4 × mass of proton

Charge of α = 2× Charge of proton.

Q. why the wave nature of matter is not observe in our daily life?.

The wavelength is extremely small and cannot be measured by any instrument.

What is de-Broglie wavelength of an e- accelocted from rest through a potential difference 100V.

Solve. V = 100 volts

m = 9.1×10^{-31} kg

$$\lambda = \frac{12.27}{\sqrt{V}} \ A^\circ$$

$$\left[\lambda = 1.227 A^\circ \right]$$

find the de-Broglie wavelength of a 15keV e^-

$K = 15 \times 10^3 \times 1.6 \times 10^{-19} J$

$$\lambda = \frac{h}{\sqrt{2mK}}$$

$$\lambda = \frac{6.623 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 15 \times 10^3 \times 1.6 \times 10^{-19}}}$$

$$\lambda = 1.0 \times 10^{-11}$$

$$\left[\lambda = 0.1 A^\circ \right]$$

Q. The knetic energy of an electron is $4.55 \times 10^{-25} J$. Calculate velocity, movementum & the wavelength of e^-

Soln. $K = \frac{1}{2} m v^2$

$$4.55 \times 10^{-25} = \frac{1}{2} \times 9.1 \times 10^{-31} \times v^2$$

$$v^2 = 10^6$$

$$\left[v = 10^3 \ m/s \right]$$

for movementum.

$$p = mv$$

$$= 9.1 \times 10^{-31} \times 10^3$$

$$= 9.1 \times 10^{-28} \ kg \ m/s$$

$$\lambda = \frac{h}{p}$$

$$= \frac{6.623 \times 10^{-34}}{9.1 \times 10^{-28}}$$

$$\left[\lambda = 7.286 \times 10^{-7} m \right]$$

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ A}^\circ$$

$$\left[\lambda = 1.227 \text{ A}^\circ \right]$$

find the de-Broglie wavelength of a 15keV e⁻

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Q. The kinetic energy of an electron is 4.55×10^{-25} J. Calculate velocity, movementum & the wavelength of e⁻

Soln.

$$K = \frac{1}{2} m v^2$$

$$4.55 \times 10^{-25} = \frac{1}{2} \times 9.1 \times 10^{-31} \times v^2$$

$$v^2 = 10^6$$

$$\left[v = 10^3 \text{ m/s} \right]$$

for movementum:

$$p = mv$$

$$= 9.1 \times 10^{-31} \times 10^3$$

$$= 9.1 \times 10^{-28} \text{ kg m/s}$$

$$\lambda = \frac{h}{p}$$

$$= \frac{6.623 \times 10^{-34}}{9.1 \times 10^{-28}}$$

$$\left[\lambda = 7.286 \times 10^{-7} \text{ m} \right]$$

Q. Can a photon and electron of the same momentum have same wavelength ? compare their wavelength if the there have the same energy.

yes

Ans The same momentum have the same wavelength.

proof

for phothon

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{hc}{c\lambda}$$

$$p_{ph} = \frac{h}{\lambda_{ph}} \quad \text{---} ①$$

for electron $\lambda = \frac{h}{p}$

$$p_e = \frac{h}{\lambda_e} \quad \text{---} ②$$

$$\because \quad p_{PH} = p_e$$

$$\frac{\cancel{h}}{\lambda_{PH}} = \frac{\cancel{h}}{\lambda_e}$$

$$\boxed{\lambda_{ph} = \lambda_e}$$

For photon.

$$E = h\nu = h\frac{c}{\lambda}$$

$$\lambda_{ph} = \frac{hc}{E} \quad \text{---} ①$$

for electron

$$\lambda_e = \frac{h}{\sqrt{2mK}} \quad \text{---} ②$$

$$\frac{\lambda_{ph}}{\lambda_e} = \frac{hc}{E} \frac{\sqrt{2mK}}{h}$$

$$\frac{\lambda_{ph}}{\lambda_e} = \frac{c}{E} \sqrt{2mK}$$

$\because K = E$

$$\frac{\lambda_{ph}}{\lambda_e} = \frac{c}{E} \sqrt{2mE}$$

$$\boxed{\frac{\lambda_{ph}}{\lambda_e} = c\sqrt{\frac{2m}{E}}}$$

imp

Q. A proton is moving with a speed of 2×10^8 m/s find the wavelength the matter wave associated with.

Solve. $v = 2\times10^8$ m/s

$m_0 = 1.67\times10^{-27}$ Kg

$\because \lambda = \frac{h}{mv}$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m = \frac{1.67\times10^{-27}}{\sqrt{1-\left(\frac{2\times10^8}{3\times10^8}\right)^2}}$$

$m = 2.24\times10^{-27}$ kg

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.623\times10^{34}}{2.24\times10^{-24}\times2\times10^8}$$

$$\boxed{\lambda = 1.47\times10^{-15}\ m}$$

Wave velocity or Phase velocity:- The wave velocity with which a singal wave traval in a medium is called wave velocity.

$$\boxed{V_p = \frac{\omega}{K}} \qquad \left[\because K = \frac{2\pi}{\lambda}\right]$$

Group velocity: The velocity with which a group of wave packet traval in the medium is callad group velocity.

$$\left[V_g = \frac{d\omega}{dK}\right]$$

Phase velocity of debrogali of matter wave:

Since $\omega = 2\pi\nu$

$$= 2\pi \frac{E}{h}$$

$$\omega = \frac{2\pi mc^2}{h} \quad \text{——} ①$$

$$\because K = \frac{2\pi}{\lambda}$$

$$K = \frac{2\pi m v}{h} \quad \text{——} ②$$

Divide eqn $\frac{①}{②}$ we get

$$\frac{\omega}{K} = \frac{\cancel{2\pi} \cancel{m} c^2}{\cancel{h}} \times \frac{\cancel{h}}{\cancel{2\pi} \cancel{m} v}$$

$$\frac{\omega}{K} = \frac{c^2}{v}$$

$$V_p = \frac{c^2}{v}$$

$$v_p \cdot v = c^2$$

$$\because v_g = v$$

$$\boxed{v_p v_g = c^2}$$

<u>Group velocity of d-brogale mattar :-</u>

<u>proof</u> Since

$$\omega = \frac{2\pi mc^2}{h}$$

$$\omega = \frac{2\pi m_0 c^2}{h\sqrt{1-\frac{v^2}{c^2}}}$$

$$\omega = \frac{2\pi m_0 c^2}{h}\left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

Diff w.r.t v we get

$$\frac{d\omega}{dv} = \frac{2\pi m_0 c^2}{h}\left(\frac{-1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(\frac{-2v}{c^2}\right)$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0 v}{h\left(1-\frac{v^2}{c^2}\right)^{3/2}} \quad \text{①}$$

$$K = \frac{2\pi mv}{h}$$

$$K = \frac{2\pi m_0 v}{h\sqrt{1-\frac{v^2}{c^2}}}$$

$$K = \frac{2\pi m_0}{h} v\left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

Diff w.r.t to 'v' get

$$\frac{dk}{dv} = \frac{2\pi m_0}{h}\left[v\left(\frac{-1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(\frac{-2v}{c^2}\right) + \left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h}\left[\frac{v^2}{c^2}\left(1-\frac{v^2}{c^2}\right)^{-3/2} + \left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$= \frac{2\pi m_0}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left[\cancel{\frac{v^2}{c^2}} + \left(1-\cancel{\frac{v^2}{c^2}}\right)\right]$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h\left(1-\frac{v^2}{c^2}\right)^{3/2}} \quad \text{—②}$$

Devide equ ①/② we get

$$\frac{d\omega}{dk} = \frac{2\pi m_0 v}{\cancel{h\left(1-\frac{v^2}{c^2}\right)^{3/2}}} \cdot \frac{h\left(1-\frac{v^2}{c^2}\right)^{3/2}}{2\pi m_0}$$

$$\frac{d\omega}{dk} = v$$

$$\boxed{v_g = v}$$

- Relation between wave velocity and Group velocity:

Proof. Since

$$v_p = \frac{\omega}{k}$$

$$\omega = v_p \cdot k$$

diff w.r.t to 'k' we get

$$\frac{d\omega}{dk} = v_p + k\frac{dv_p}{dk}$$

$\cancel{v_g = v_g}$ $\quad v_g = v_p + k \frac{d\lambda}{dk} \frac{dv_p}{d\lambda}$

$$\because k = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{2\pi}{k}$$

$$\therefore \frac{d\lambda}{dk} = \frac{-2\pi}{k^2}$$

$$v_g = v_p + k\left(\frac{-2\pi}{k^2}\right)\frac{dv_p}{d\lambda}$$

$$v_g = v_p - \left(\frac{2\pi}{k}\right)\frac{dv_p}{d\lambda}.$$

$$\boxed{v_g = v_p - \lambda \frac{dv_p}{d\lambda}}$$ in dispersive medium

In Non-dispersive medium $[v_p = \text{costant}]$

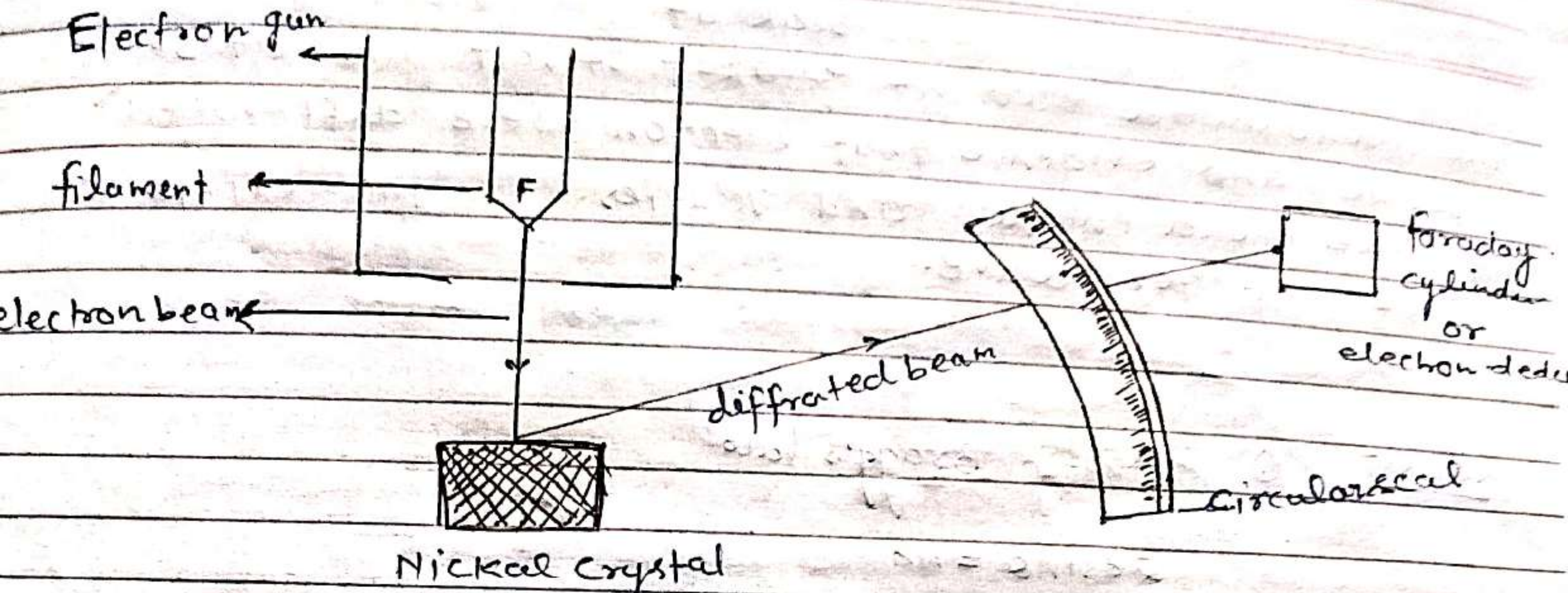
$$v_g = v_p - 0$$

$$[v_g = v_p]$$

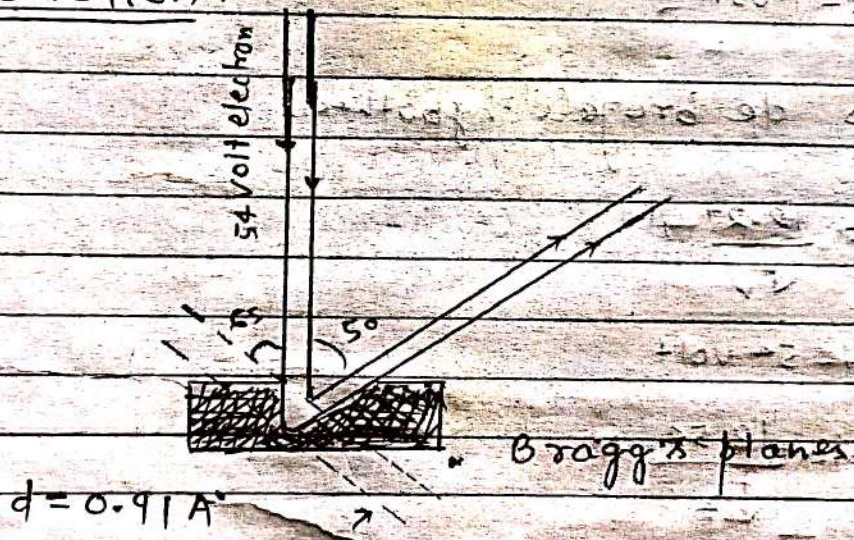
Davisson and Germer Experiment:-

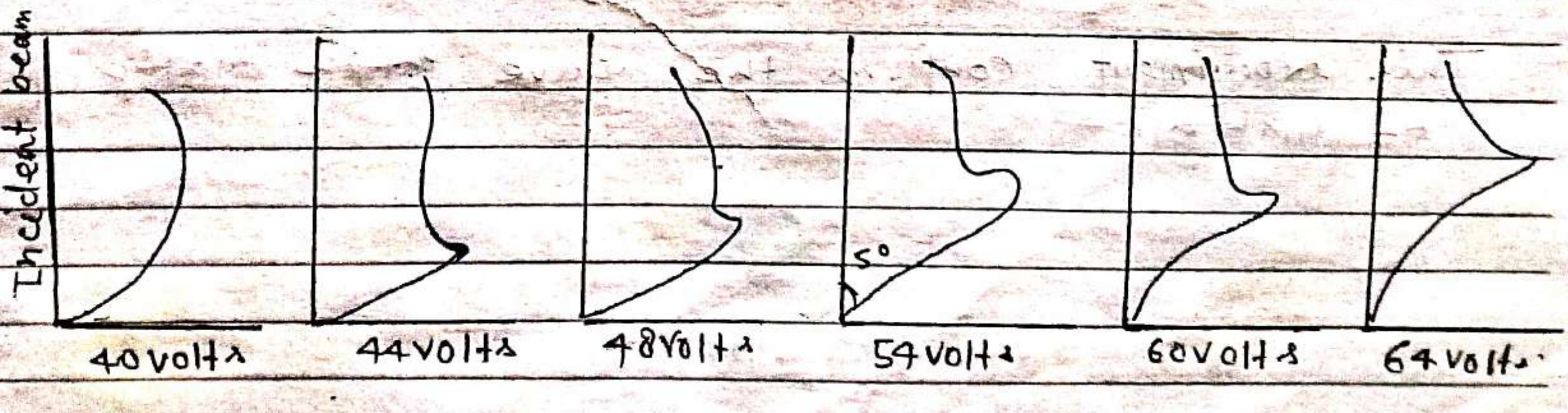
Object: Davisson and Germer Experiment conclude that eletron have wave properties.

Experimental set up:-



Observation:





The pronounced current ~~peakat~~ peak at 54 volt and at 50°
provide and evidence that electron were diffracted
by the nickel crystal and verifies. the existance
of electron wave.

<u>Result</u> ① Apply Bragg's law

$$2d\sin\theta = n\lambda$$

$d = 0.91\,A°$ $\theta = 65°$ $n = 1$

$$2 \times (0.91) \times \sin 65 = 1 \times \lambda$$

$$\boxed{\lambda = 1.65\,A°}$$

② According to de brogali hypothesis

$$\lambda = \frac{12.27}{\sqrt{v}}\,A°$$

$$V = 54\text{ volt}$$

$$\lambda = \frac{12.54}{\sqrt{54}}\,A°$$

$$\boxed{\lambda = 1.67\,A°}$$

This experiment confirms the wave particle duality
of matter.

Note: The eqn of motion of matter waves can expressed as

1) $\psi = \psi_0 \sin\omega t$
2) $\psi = \psi_0 \sin(\omega t + \phi)$
3) $\psi = \psi_0 e^{-i\omega t}$
4) $\psi = A\sin kx + B\cos kx$
5) $\psi = A\sin\theta + B\cos\theta$

* The classical differential eqn of wave motion

$$\boxed{\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}}$$ where $\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$

Wave function (ψ): The quantity whose periodic variation characteries matter wave is called wave functio

② physical significance of wave function: The wave function is related to the probability of finding the particle have ~~that~~ point and ~~at~~ at that instant time

① It must be finite everywhere.
② It must be single value.
③ It must be continious.
④ It must be normilized.

$$\boxed{\int_{-\infty}^{\infty} |\psi w|^2 \, dx\,dy\,dz = 1}$$

Schrodinger wave equation: There are two types

① Time independent schrodinger wave equation:

proof from classical differential wave ~~motion~~ equation is

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{--- ①}$$

The general sol/n of wave motion is

$$\psi = \psi_0 \sin\omega t \quad \text{--- ②}$$

diff. w.r.t to t we get

$$\frac{\partial\psi}{\partial t} = \omega\psi_0 \cos\omega t$$

again

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi_0 \sin\omega t$$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi$$

$$= -(2\pi\nu)^2\psi$$

$$= -\left(\frac{2\pi v}{\lambda}\right)^2\psi$$

$$\frac{\partial^2\psi}{\partial t^2} = -\frac{4\pi^2 v^2\psi}{\lambda^2} \quad \text{--- ③}$$

from eq① & ③ we get

$$\nabla^2\psi = \frac{1}{v^2}\left(-\frac{4\pi^2 v^2}{\lambda^2}\right)\psi$$

$$\left[\ \nabla^2\psi = -\frac{4\pi^2}{\lambda^2}\psi\ \right]$$

$$\nabla^2\psi = -\frac{4\pi^2}{\lambda^2}\psi$$

$$\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0 \qquad \left(\because \lambda = \frac{h}{mv}\right)$$

$$\nabla^2\psi + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0 \quad \text{—— (4)}$$

Since total Energy $E = K + V$

$$K = E - V$$

$$\frac{1}{2}mv^2 = E - V$$

$$m^2v^2 = 2m(E-V) \quad \text{—— (5)}$$

from eqn (4) & (5) we get

$$\nabla^2\psi + \frac{4\pi^2 \cancel{\times} 2m(E-V)}{h^2}\psi = 0$$

$$\boxed{\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0}$$

for ~~freee~~ free particle $V = 0$

$$\boxed{\nabla^2\psi + \frac{8\pi^2 m}{h^2}E\psi = 0}$$

$$\hbar = \frac{h}{2\pi}$$

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}E\psi = 0}$$

for numerical value.

$$E_n = \frac{n^2h^2}{8mL^2} \qquad n = 1, 2, 3$$

(ii) $\psi_n = \sqrt{2/L}\,\sin\frac{n\pi x}{L}$

(iii) $\int_{-\infty}^{\infty} |\psi_n(x)|^2 dx = 1$

$$P = \int_{x_1}^{x_2} |\psi_n(x)|^2 dx$$

$$P = |\psi_n(x)|^2 \Delta x$$

(vi) $\langle x \rangle = \int_{-\infty}^{\infty} x|\psi_n(x)|^2 dx$

Time depended schrodinger wave equations:

Proof: from classical differential eqn of wave motion is

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{--- (1)}$$

The general soln of wave motion is

$$\psi = \psi_0 e^{-i\omega t}$$

Diff w.r.t 't' we get

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0 e^{-i\omega t}$$

$$= -i(2\pi\nu)\psi$$

$$\frac{\partial\psi}{\partial t} = -i\left(2\pi\frac{E}{h}\right)\psi$$

$$\frac{-h}{i2\pi}\frac{\partial\psi}{\partial t} = E\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi \quad \text{--- (2)}$$

Schrodinger Time-independent wave eqn for free particle

$$\nabla^2\psi + \frac{2m}{\hbar^2}E\psi = 0$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E\psi) = 0$$

from eqn (2) & (3) we get

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left[i\hbar\frac{\partial\psi}{\partial t}\right] = 0$$

Time dependent Schrodinger wave equation:

Proof: from classical differential eqn of wave motion is

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{--- (1)}$$

The general soln of wave motion is

$$\psi = \psi_0 e^{-i\omega t}$$

Diff w.r.t 't' we get

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0 e^{-i\omega t}$$

$$= -i(2\pi\nu)\psi$$

$$\frac{\partial\psi}{\partial t} = -i\left(2\pi\frac{E}{h}\right)\psi$$

$$\frac{-h}{i2\pi}\frac{\partial\psi}{\partial t} = E\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi \quad \text{--- (2)}$$

or $\left(\frac{-\hbar^2}{2m}\nabla^2 + \ldots\right)$

$\boxed{H\psi = E\psi}$

$H = H\psi$

Schrodinger Time independet wave equation

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E\psi - V\psi) = 0 \quad \text{--- (3)}$$

from eqn (2) & (3) we get

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left[i\hbar\frac{\partial\psi}{\partial t} - V\psi\right] = 0$$

$$\nabla^2\psi = -\frac{2m}{\hbar^2}\left[i\hbar\frac{\partial\psi}{\partial t} - V\psi\right]$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi = i\hbar\frac{\partial\psi}{\partial t} - V\psi$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\left[H\psi = E\psi\right]$$

E = Energy operator

H = Hamiltonian operator

Particle in one-dimensional potential box;

or

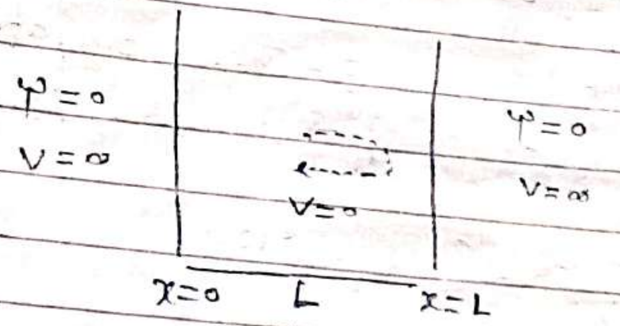
Infinite square well

or

Infinite potential box

Deep infinite potential box.

Sol^n



The potential function is defined as

$$V(x) = \begin{cases} \infty & 0 \geq x \geq L \\ 0 & 0 \leq x \leq L \end{cases} \quad \text{—— (1)}$$

The schrodinger time-independent wave eq^n

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$$

for free particle $V = 0$

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2} E\psi = 0 \quad \text{—— (2)}$$

let $\frac{8\pi^2 m}{h^2} E = k^2$ —— (3)

Also $\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0$

The genral sol^n of eq^n (4) is

$\psi(x) = A\sin kx + B\cos kx$ —— (5)

Applying first boundary condition

$x = 0$ & $\psi = 0$

$0 = A\sin k0 + B\cos k0$

$[B = 0]$

Applying second boundary condition

$x = L$ & $\psi = 0$

$0 = A\sin kL + 0$

$A\sin kL = 0$

$\sin kL = 0 \quad A \neq 0$

$\sin kL = \sin n\pi \quad (n = 0, 1, 2, 3, ---)$

$kL = n\pi$

$\left[k = \frac{n\pi}{L}\right]$ —— (6)

from eqn (3) & (6) we get

$$\frac{8\pi^2 m}{h^2} E = \frac{n^2\pi^2}{L^2}$$

$$\left[E = \frac{n^2h^2}{8mL^2}\right]$$

This energy is know as Eigan value

from eqn (3) & (6) we get

$$\left[\psi(x) = A\sin\frac{n\pi x}{L}\right]$$

This is known as Eigan function.

Normalisation of wave form function.

To find the value of constant A)

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = 1$$

$$\int_0^L \left| A \sin \frac{n\pi L}{L} \right|^2 dx = 1$$

$$A^2 \int_0^L \sin^2 \frac{n\pi x}{L} \, dx = 1$$

$$A^2 \int_0^L \frac{1}{2} \left[1 - \cos \frac{2n\pi x}{L} \right] dx = 1$$

$$\frac{A^2}{2} \left[x - \frac{L}{2n\pi} \sin \frac{2n\pi x}{L} \right]_0^L = 1$$

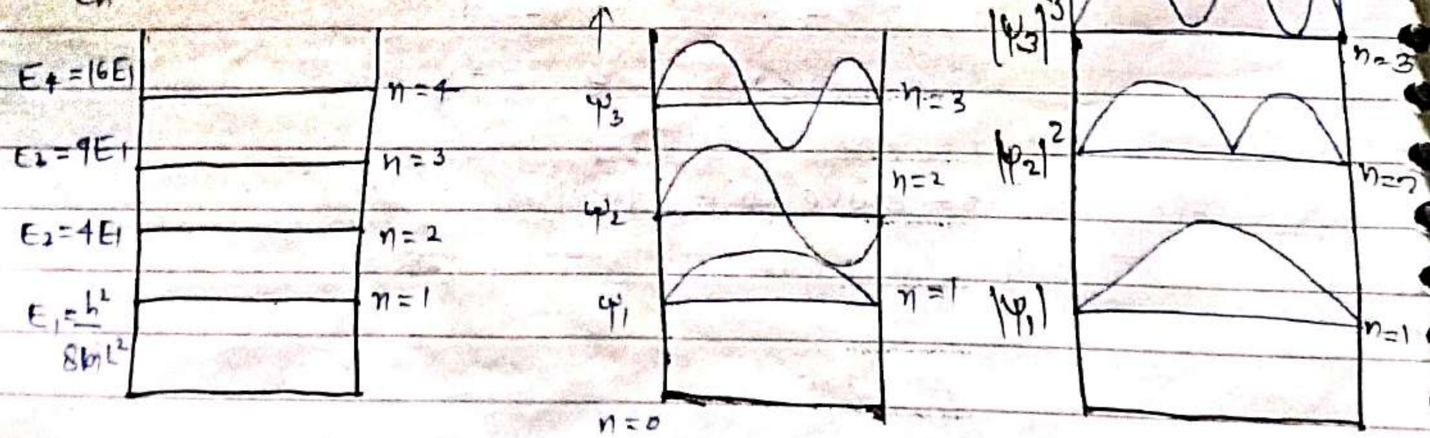
$$\frac{A^2}{2} [L] = 1$$

$$A^2 = \frac{2}{L}$$

$$\boxed{A = \sqrt{\frac{2}{L}}}$$

The ~~waveform~~ wavefunction becomes

$$\boxed{\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}}$$



Q. An electron is bound in one dimentional potential for which has width 2.5×10^{-10} m. Assuming the height of the box to be infinity calculate the lowest to permitted energy value of the electron

Solve. $L = 2.5 \times 10^{-10}$ m , $n = 1$

$$E_n = \frac{n^2h^2}{8mL^2}$$

$$E_1 = \frac{(1)^2 \times (6.628 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2.5 \times 10^{-10})^2}$$

$$E_1 = 9.640 \times 10^{-19} \text{ Joule}$$

$$E_2 = \frac{(2)^2 \times (6.623 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2.5 \times 10^{-10})^2}$$

$$\boxed{E_2 = 9.256 \times 10^{-18} \text{ Joule}}$$

Q.4. Calculate the energy difference between the ground state and the first excited state for an electron in a one dimensional rigid box of length 10^{-8} cm.

Sol^n $n = 1$ $n = 2$ $L = 10^{-8}$ cm $= 10^{-10}$ m

$$E_n = \frac{n^2h^2}{8mL^2}$$

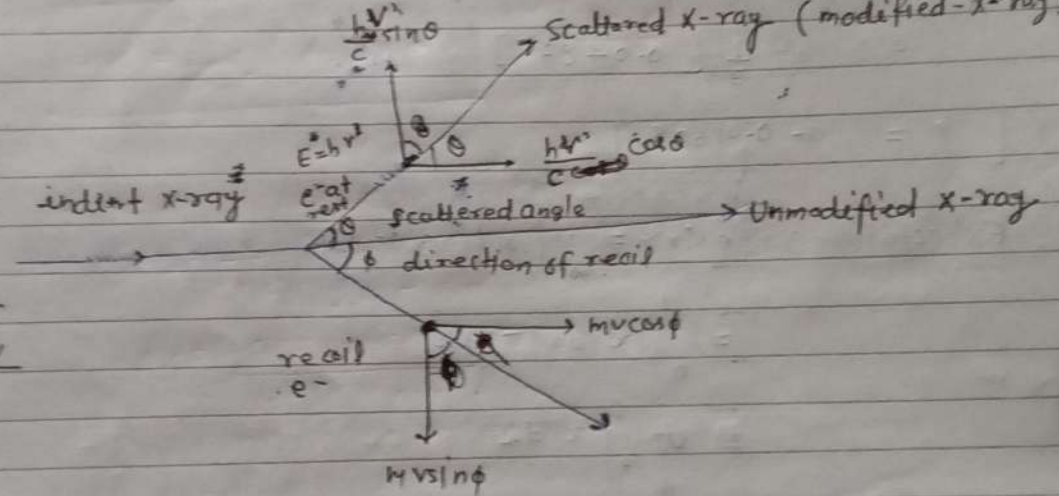
$$E_1 = \frac{(1)^2 \times (6.623 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$$= \frac{6.025 \times 10^{-28} \text{ J}}{1.6 \times 10^{-19}}$$

$$E_1 = 3.765 \times 0 \text{ eV}$$

Compton Effect :- The change in frequency or wave length due to scattering is called compton effect

$$\left[\ \Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)\ \right] \qquad \Delta\lambda = \text{compton shift}$$



$E = hv$

$p = \frac{hv}{E}$

According to the principle of convertation of energy

$$hv + m_0c^2 = hv' + mc^2 \quad \text{--- (1)}$$

$$h(v - v') + m_0c^2 = mc^2$$

$$h^2(v^2 + v'^2 - 2vv') + m_0^2c^4 + 2h(v - v')m_0c^2 = m^2c^4 \quad \text{--- (2)}$$

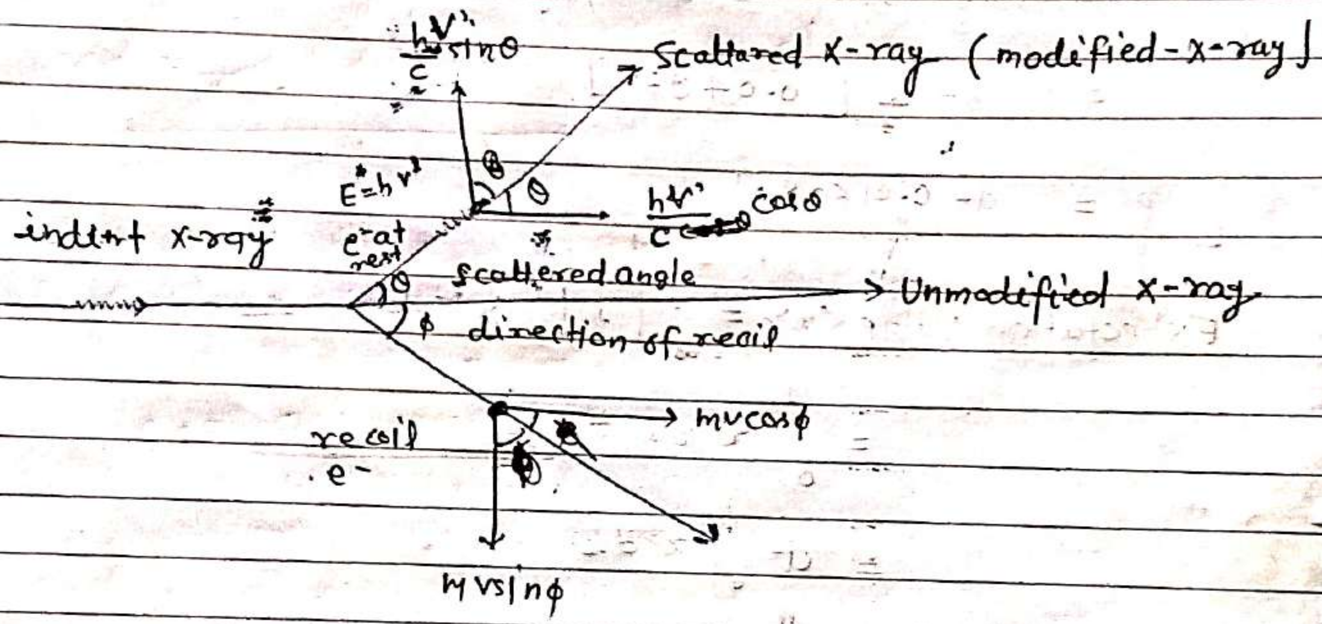
According to the principle of conversion of momentum

$$\frac{hv}{c} + 0 = \frac{hv'}{c}\cos\theta + mv\cos\phi \quad \text{--- (3)}$$

$$\frac{hv}{c} - \frac{hv'}{c}\cos\theta = mv\cos\phi$$

Compton Effect :- The change in frequency or wave length due to st scattering is called compton effect

$$\left[\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta) \right] \quad \Delta\lambda = \text{Compton shift}$$



$E = hv$

$p = \frac{hv}{c}$

According to the principle of convertation of energy

$$hv + m_0c^2 = hv' + mc^2 \quad \text{—①}$$

$$h(v - v') + m_0c^2 = mc^2$$

$$h^2(v^2 + v'^2 - 2vv') + m_0^2c^4 + 2h(v - v')m_0c^2 = m^2c^4 \quad \text{—②}$$

According to the principle of conversion of movement -

$$\frac{hv}{c} + 0 = \frac{hv'}{c}\cos\theta + mv\cos\phi \quad \text{—③}$$

$$\frac{hv}{c} - \frac{hv'}{c}\cos\theta = mv\cos\phi$$

$h\nu - h\nu'\cos\theta = \cancel{m v c o} \; \cancel{m v c o} \; m v c\cos\phi$

$h^2\nu^2 + h^2\nu'^2\cos^2\theta - 2h^2\nu\nu'\cos\theta = m^2v^2c^2\cos^2\phi$ —④

$0 + 0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi$ —⑤

(y-direction)

$\frac{h\nu'}{c}\sin\theta = mv\sin\phi$

$h\nu'\sin\theta = mvc\sin\phi$

$h^2\nu'^2\sin^2\theta = m^2v^2c^2\sin^2\phi$ —⑥

Addition of ④ + ⑥ we get

$h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\theta = m^2v^2c^2$ —⑦

subtracting eqn ⑦ from eqn ③ we get

$-2h^2\nu\nu' + 2h^2\nu\nu'\cos\theta + m_0^2c^4 + 2h(\nu-\nu')m_0c^2 = m^2c^4 - m^2v^2c^2$

$= m^2c^2(c^2-v^2)$

$= \frac{m_0^2c^2}{\left[1-\frac{v^2}{c^2}\right]}(c^2-v^2)$

$= \frac{m_0^2c^4}{\cancel{(c^2-v^2)}}\cancel{(c^2-v^2)}$

$-2h^2\nu\nu' + 2h^2\nu\nu'\cos\theta + \cancel{m_0^2c^4} + 2h(\nu-\nu')m_0c^2 = \cancel{m_0^2c^4}$

$-2h^2\nu\nu' + 2h^2\nu\nu'\cos\theta + 2h(\nu-\nu')m_0c^2 = 0$

$\cancel{2h}(\nu-\nu')m_0c^2 = \cancel{2h^2}\nu\nu'(1-\cos\theta)$

$\left(\frac{\nu-\nu'}{\nu\nu'}\right) = \frac{h}{m_0c^2}(1-\cos\theta)$

$$\frac{1}{v'} - \frac{1}{v} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\frac{c}{v'} - \frac{c}{v} = \frac{hc}{m_0c^2}(1-\cos\theta)$$

$$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)$$

$$\boxed{\Delta\lambda = \frac{h}{m_0c}(1-\cos\theta)}$$ Proved

Case ① if $\theta = 0$

$$\Delta\lambda = \frac{h}{m_0c}(1-\cos\theta)$$

$$\Delta\lambda = \frac{6.622\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos 0)$$

$$[\Delta\lambda = 0]$$

Case (II) if $\theta = 90$

$$\Delta\lambda = \frac{6.23\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos 90)$$

$$\Delta\lambda = 0.02426\times10^{-10}\,m$$

$$[\Delta\lambda = 0.02426\,A^\circ]$$

III) if $\theta = 180^\circ$

$$\Delta\lambda = \frac{6.623\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos 180)$$

$$[\Delta\lambda = 0.04852\,A^\circ]$$

Unmodified x-rays :- The radiation of Unchange frequency in the scatterd beam is known as Unmodified x-rays.

Modified x-rays :- The radiation of lower frequency or higer wavelength due to scatting is called modified x-rays.

Q. In compton scattering the incident proton higer wavelength 3×10^{-10} m. Calculate the wavelength of scattered radiation if they are ~~bwde~~ veiwed at angle of 60° to the direction of incident.

Solⁿ: $\lambda = 3\times10^{-10}$ m $\quad \theta = 60$

$$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)$$

$$\lambda' = \lambda + \frac{h}{m_0c}(1-\cos\theta)$$

$$\lambda' = 3\times10^{-10} + \frac{6.623\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos60)$$

$$= 3\times10^{-10} + 1.213\times10^{-12}$$

$$= 3\times10^{-10} + 0.012\times10^{-10}$$

$$= 3.012\times10^{-10}\text{ m}$$

$$= 3.012\ A^{\circ}\ \text{Ans}$$

Q. In a compton experiment the wavelength of X-ray radiation is scattered at an angle of 45 is 0.22 A°. Calcu the wavelength of incident x-rays.

Soln. $\theta = 45$ $\lambda = 0.22$ A°

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

$$\lambda' = \lambda + \frac{h}{m_0 c} \cdot (1 - \cos\theta)$$

$$= 0.22 + \frac{6.623 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}} (1 - \cos 45)$$

$$= 0.22 + 7.10 \times 10^{-13}$$

$$= 0.22 \times 10^{-10} + 7.10 \times 10^{-13}$$

$$= 0.022 \times 10^{-10} + 7.10 \times 10^{-10}$$

Q. Calculate compton shift if X-ray of $\lambda = 1$A° are scattered from a carbon block. The scattered radiation viewed at 90 to the incident beam.

$\lambda = 1$ A° $\theta = 90$

$$\Delta\lambda = \frac{6.623 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}} (1)$$

=

Short type Answers.

Q. Explain basic postulates of Plank's law of radiation.

Ans: The basic postulates of Plank's law of radiation are

1. Energy is ~~contilses~~. quantized.
2. Energy is propotional to frequency.
3. Radiation is emmited in packets
4. Radiation emission depend on temprature.

Q3. Can compton effect be observed with visible light.

Ans. No, the compton effect is not usally observed with visible light because the energy of visible light photons is not high.

Q. Write down Plank's expression for special energy density in Black Body radiation.

Ans. A black body energy density between λ and $\lambda + d\lambda$ is the energy of mode $E = hc/\lambda$ times the density of photon states.

Q10. Give the interpretation of Bohr's quantization rule.

Ans. Bohr's quantization rule state that electron and H atom can only orbit the nucleus in certain orbit, with specic angular momentum values

Ans: Modified x-ray have a longer wavelength than unmodified x-ray. The unmodified x-ray have the same wavelenght as the orginal x-ray beam.

Ans:

Because the concept was the matter wave as describe by the d-Brogali hypothesis directly relate the wavelenght of a wave to the momentum a particle which is only present when the particle is moving.