Unit 02

(Electro magnetic theory)

E = Electric field intensity
H = magnetic field intensity
D = Electric flux density (Displacement vector)
B = magnetic flux Density
ρ = charge density

V = Volume
σ = Conductivity of medium
J = Convention current density
J_c = conductance " "
J_D = displacement current density

ϵ = Permitivity of medium.
ϵ_0 = permittivity of free space $8.85 \times 10^{-12} C^2/Nm^2$
μ = permeability of medium
μ_0 = permeability of free space $4\pi \times 10^{-7}$ h/m

ϵ_0 = Relative permeability
μ_r = Relative permeablity.

(I) $I = \frac{Q}{t}$ (II) $J = \frac{I}{A}$ (III) $D = \sigma = \frac{q}{A}$

(IV) $D = \epsilon E$ ~~$D = \epsilon E$~~ (V) $B = \mu H$ (VI) $J = \sigma E$ (VI) $J = \rho_v V$

(XI) $\epsilon = \epsilon_0 \epsilon_r$ (X) $\mu = \mu_0 \mu_r$ (XII) $\frac{J_c}{J_d} = \frac{\sigma}{\omega \epsilon}$

(XII) $J_T = J_c + J_d$

Vector identity.

(i) $\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2 E$

(ii) $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{E}) = 0$

* Gauss's divergence theorem.

$$\oint_s E\,ds = \int_v (\nabla E)\,dv$$

&

$$\oint B\cdot ds = \int_v (\nabla\cdot B)\,dv$$

* Stoke's therom

$$\oint E\cdot dl = \oint(\nabla\times E)\,ds$$

or $$\oint B\,dl = \oint_s (\nabla\times B)\,ds$$

- Electrostatic fields.

		Diff. from	Inter from
(I)	Gauss' law	$\vec{\nabla}D = \rho_v$	$\oint D\,ds = q_{enc}$
(II)	Fareday's law	$\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$	$\oint E\,dl = \frac{\partial B}{dt}$

magnetostatic field:

(I)	Gauss law	$\vec{\nabla}\bar{B} = 0$	$\oint_s \bar{B}\,ds = 0$
(II)	Ampear's law	$\vec{\nabla}\times\vec{H} = J$	$\oint \vec{H}\,dl = I_{enc}$

maxwell's Equations.

* Diff form (I) $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon}$ / $\vec{\nabla}D = \rho$

(ii) $\vec{\nabla} B = 0$ (iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$

(iv) $\vec{\nabla} \times \vec{H} = J + \frac{\partial D}{\partial t}$ / $\vec{\nabla} \times \vec{B} = \mu \left[J + \frac{\partial D}{\partial t} \right]$

* Integral form

$$\oint E ds = \frac{1}{\varepsilon} \int_v \rho dv$$

$$\oint B ds = 0$$

$$\oint E dl = -\oint_s \left(\frac{\partial B}{\partial t} \right) ds$$

$$\oint H \cdot dl = \oint_s \left(J + \frac{\partial D}{\partial t} \right) d \cdot s$$

• Displacement current: According to the modification of Ampere's law changing electric field is equivalent to current is called displacement current.

$$\left[I_d = A \frac{\partial D}{\partial t} \right] \quad \& \quad \left[J_d = \frac{\partial D}{\partial t} \right]$$

* modification of Ampere's law using the idea of displacement current.

Proof According to Ampere's law.

$$\vec{\nabla} \times \vec{B} = \mu_0 J \quad —① $$

Taking div on both side we get

$$\vec{\nabla} (\vec{\nabla} \times \vec{B}) = \mu_0 (\vec{\nabla} \cdot \vec{J})$$

$$\vec{\nabla} \cdot J = 0 \quad —②$$

this equation is in contradiction of equation of continuity.

$$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0 \quad \text{--- (3)}$$

So, maxwell's concluded that equation (1) is incomplete and tried to modify.

$$\vec{\nabla} \times B = \mu_0 J + \text{something} \quad \text{--- (4)}$$

According to Gauss's law.

$$\vec{\nabla} \cdot \vec{D} = \rho$$

Diff w.r.t to 't' we get

$$\vec{\nabla} \cdot \frac{\partial D}{\partial t} = \frac{\partial \rho}{\partial t}$$

Adding $\vec{\nabla} \cdot J$ on both side we get

$$\vec{\nabla} \cdot J + \vec{\nabla} \cdot \frac{\partial D}{\partial t} = \nabla \cdot J + \frac{\partial \rho}{\partial t}$$

$$\nabla \left(J + \frac{\partial D}{\partial t} \right) = 0 \quad \text{--- (5)}$$

Thus. $\vec{\nabla} \cdot J = 0$ for steady state current

$\nabla \left(J + \frac{\partial D}{\partial t} \right) = 0$ for everywhere.

In this way, maxwell replace J in Amperes law

$$\vec{J} \rightarrow \left(\vec{J} + \frac{\partial D}{\partial t} \right)$$

So, $\left[\vec{\nabla} \times B = \mu_0 \left(\vec{J} + \frac{\partial D}{\partial t} \right) \right]$

Continuity equation: The mathematical represtion of the law of conservation of charge in differential form is called equation of continuity.

formula : $\left[\vec{\nabla}\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0\right]$

proof. from maxwell's fourth eqn

$$\vec{\nabla}\times\vec{B} = \mu\left[J + \epsilon\frac{\partial E}{\partial t}\right]$$

taking div on both side we get

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu\left[\vec{\nabla}\vec{J} + \epsilon\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E})\right]$$

$$0 = \mu\left[\vec{\nabla}\cdot J + \epsilon\frac{\partial}{\partial t}(\nabla\cdot E)\right]$$

$$0 = \vec{\nabla}\cdot J + \epsilon\frac{\partial}{\partial t}\left(\frac{\rho}{\epsilon}\right) \qquad \nabla\cdot\vec{E} = \frac{\rho}{\epsilon}$$

$$\left[\nabla\cdot J + \frac{\partial \rho}{\partial t} = 0\right]$$ proved

- Derivation of maxwell's equation:

① maxwell's first eqn:

According to Gauss's law

$$\oint_c E d\vec{s} = \frac{q}{\epsilon} \quad \text{——①}$$

if the charge q inside the closed surface is

$$q = \int_v \rho dv \quad \text{——②}$$

from ① & ② we get

$$\left[\oint E\,ds = \frac{1}{\epsilon}\int \rho\,dv\right] \quad \text{--- ③}$$

This is integral for of maxwell first eqⁿ

using Gauses diverge

$$\oint E\,ds = \int_v (\vec{\nabla}\cdot\vec{E})\,dv \quad \text{--- ④}$$

from eqn ③ & ② we get

$$\int(\vec{\nabla}\vec{E})\,dv = \frac{1}{\epsilon}\int \rho\,dv$$

comparing above eqn

$$\left[\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon}\right]$$

or

$$\left[\vec{\nabla}\cdot\vec{D} = \rho\right]$$

② maxwell second eqn.

According to Gauss law

$$\phi_B = 0$$

$$\oint_S E\,d\bar{s} = 0 \quad \text{--- ①}$$

This is integral for maxwell second equation.

using Gauss' divergence therom.

$$\oint \vec{B}\,d\vec{s} = \int(\vec{\nabla}\cdot\vec{B})\,dv \quad \text{--- ②}$$

from eqn ① & ② we get

$$\int(\vec{\nabla}\vec{B})\,dv = 0 \quad \left[\vec{\nabla}\cdot\vec{B} = 0\right]$$

③ Maxwell's third equation:

According to faraday law of electro magnetic induction.

$$e = -\frac{\partial \phi_B}{\partial t} \quad \text{—①}$$

The work done in moving a positive charge around a closed circuit is equal to electromotive force.

$$e = \oint_c \vec{E} dl \quad \text{—②}$$

The magnetic flux through a circuit is

$$\phi_B = \int \vec{B} ds \quad \text{—③}$$

from equation ① ② & ③

$$\oint_c \vec{E} \cdot dl = -\oint_s \left(\frac{\partial B}{\partial t}\right) ds \quad \text{—④}$$

This is integral from of maxwell third equation.

using stokes therom

$$\oint_c E dl = \oint_s (\vec{\nabla} \times E) ds \quad \text{—⑤}$$

from eqn ④ & ⑤ we get

$$\oint (\vec{\nabla} \times \vec{E}) ds = \oint \left(\frac{\partial B}{\partial t}\right) ds$$

Comparing

$$\boxed{\vec{\nabla} \cdot \vec{E} = -\frac{\partial B}{\partial t}}$$

Ans

④ Maxwell's fourth equation: According to ampear's law

$$\oint_c \bar{B}\cdot dl = \mu I \quad \text{——(i)}$$

$$\oint B\cdot dl = \mu (I_c + I_d) \quad \text{——(ii)}$$

$$\because I_c = \oint J_c \, ds \quad \& \quad I_d = \oint J_d \, ds \quad \text{——(iii)}$$

from (ii) & (iii) we get

$$\oint B\cdot dl = \mu \left[\oint J_c ds + \int_s J_d ds \right]$$

$$\oint_c B\cdot dl = \mu \int_s (J_c + J_d)\, ds$$

$$\because J_c = J \quad \& \quad J_d = \frac{\partial D}{\partial t}$$

$$\oint B\cdot dl = \mu \oint_s \left[J + \frac{\partial D}{\partial t} \right] ds \quad \text{——(iv)}$$

This integral from of maxwell fourth eqn

using stokes therom.

$$\oint B\cdot dl = \int_s (\vec{\nabla} \times \bar{B})\, ds \quad \text{——(v)}$$

from eqn (iv) & (v) we get

$$\int (\nabla \times \bar{B}) ds = \mu \int \left[J + \frac{\partial D}{\partial t} \right] ds \quad \text{——(vi)}$$

comparing

$$\vec{\nabla} \times \bar{B} = \mu \left[J + \frac{\partial D}{\partial t} \right]$$

* Maxwell's eqn in conducting and non-conducting medium

(1) $\nabla \cdot E = \frac{\rho}{\epsilon}$

(2) $\nabla \cdot B = 0$

(3) $\nabla \times E = -\mu \frac{\partial H}{\partial t}$

(4) $\nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t}$

(1) wave Eq in form of E

$\because \vec{\nabla} \times \vec{E} = -\mu \frac{\partial H}{\partial t}$

taking curl on both side we get

$\nabla \times (\nabla \times E) = -\mu \frac{\partial}{\partial t} (\nabla \times H)$

$\vec{\nabla}(\vec{\nabla} \cdot E) - \nabla^2 E = -\mu \frac{\partial}{\partial t} \left[\sigma E + \epsilon \frac{\partial E}{\partial t} \right]$

Since there is no charge within an conductor because the recharge reside on the surface of the conductor.

$\rho = 0$

$\vec{\nabla} \cdot E = 0$

$\vec{\nabla}(0) - \nabla^2 (E) = -\mu \frac{\partial}{\partial t} \left[\sigma E + \epsilon \frac{\partial E}{\partial t} \right]$

$-\nabla^2 E = -\mu \sigma \frac{\partial E}{\partial t} - \mu \epsilon \frac{\partial^2 E}{\partial t^2}$

$$\boxed{\nabla^2 E = \mu \sigma \frac{\partial E}{\partial t} + \mu \epsilon \frac{\partial^2 E}{\partial t^2}}$$

In case of non-conducting medium ($\sigma = 0$)

$$\boxed{\nabla^2 E = \mu \epsilon \frac{\partial^2 E}{\partial t^2}}$$

wave equation in term of H -

from ①

$$\vec{\nabla} \times H = \sigma E + \epsilon \frac{\partial E}{\partial t}$$

Taking curl on both side

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{H}) = \sigma(\nabla \times E) + \epsilon \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{E})$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{H}) - \nabla^2 H = \sigma\left(-\mu \frac{\partial H}{\partial t}\right) + \epsilon \frac{\partial}{\partial t}\left(-\mu \frac{\partial H}{\partial t}\right)$$

$$\nabla(0) - \nabla^2 H = -\mu\sigma \frac{\partial \vec{H}}{\partial t} - \mu\epsilon \frac{\partial^2 H}{\partial t^2}$$

$$-\nabla^2 H = -\mu\sigma \frac{\partial H}{\partial t} - \mu\epsilon \frac{\partial^2 H}{\partial t^2}$$

$$\boxed{\nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} + \mu\epsilon \frac{\partial^2 H}{\partial t^2}}$$

In case of non-conducting $\sigma = 0$

$$-\nabla^2 E = -\mu\sigma \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

In case of non conducting medium $\sigma = 0$

$$\boxed{\nabla^2 E = \mu\epsilon \frac{\partial^2 E}{\partial t^2}}$$

Similarly

$$\boxed{\nabla^2 B = \mu\sigma \frac{\partial B}{\partial t} + \mu\epsilon \frac{\partial^2 B}{\partial t^2}}$$

&

$$\boxed{\nabla^2 D = \mu\sigma \frac{\partial D}{\partial t} + \mu\epsilon \frac{\partial^2 D}{\partial t^2}}$$

In case of non-conducting $\sigma = 0$

$$\boxed{\nabla^2 B = \mu \epsilon \frac{\partial^2 B}{\partial t^2}}$$

& $$\boxed{\nabla^2 D = \mu \epsilon \frac{\partial^2 D}{\partial t^2}}$$

- **Maxwell's Eqn in free space:**

Since, In a vaccum $\rho = 0$ & $J = 0$

① $\vec{\nabla} \cdot \vec{E} = 0$ ② $\vec{\nabla} \cdot \vec{B} = 0$

③ $\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$ ④ $\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$

① wave eqn in term of $\bar{E}$:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$$

taking curl on Both side

$$\bar{\nabla} \times (\vec{\nabla} \times \vec{E}) = \frac{-\partial (\bar{\nabla} \times \bar{B})}{\partial t}$$

$$\bar{\nabla}(\bar{\nabla} \cdot \bar{E}) - \nabla^2 E = \frac{-\partial (\bar{\nabla} \times \bar{B})}{\partial t}$$

$$0 - \nabla^2 E = \frac{-\partial}{\partial t} \left(\mu_0 \epsilon_0 \frac{\partial E}{\partial t} \right)$$

$$0 - \nabla^2 E = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t}$$

$$\boxed{\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

② wo

② wave equation in term of $\vec{H}$:-

since

$$\nabla \times \vec{H} = \varepsilon_0 \frac{\partial E}{\partial t}$$

Taking curl on both side.

$$\nabla \times (\nabla \times \vec{H}) = \varepsilon_0 \frac{\partial}{\partial t}(\nabla \times E)$$

$$\nabla (\nabla \cdot H) - \nabla^2 H = \varepsilon_0 \frac{\partial}{\partial t} \frac{\partial B}{\partial t} \quad \varepsilon_0 \frac{\partial}{\partial t}\left(\frac{-\partial B}{\partial t}\right)$$

$$0 - \nabla^2 H = -\varepsilon_0 \frac{\partial^2 B}{\partial t}$$

$$\nabla^2 H = \varepsilon_0 \frac{\partial^2 B}{\partial^2 t}$$

$$\boxed{\nabla^2 H = \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2}}$$

Similarly $\boxed{\nabla^2 B = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}}$

& $\boxed{\nabla^2 D = \mu_0 \varepsilon_0 \frac{\partial^2 D}{\partial t^2}}$

So Standard form of wave eqn

$$\nabla^2 \begin{bmatrix} E \\ H \\ B \\ D \end{bmatrix} = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \begin{bmatrix} E \\ H \\ B \\ D \end{bmatrix}$$

$$\boxed{\nabla^2 A = \mu_0 \varepsilon_0 \frac{\partial^2 A}{\partial t^2}}$$

or $\boxed{\nabla^2 A = \frac{1}{V^2} \frac{\partial^2 A}{\partial t^2}}$

where $\frac{1}{v^2} = \mu_0\epsilon_0$ Here v is the velocity of EM waves

$$v = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$V = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}}$$

$$V = 2.99 \times 10^8 \ m/s$$

$[V = c]$

we can write

$$\left[c = \frac{1}{\sqrt{\mu_0\epsilon_0}} \right]$$

Transverse nature of EM wave:- The wave equation for E and H are

$$\left.\begin{aligned} \nabla^2 E &= \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \\ \& \quad \nabla^2 H &= \frac{1}{c^2}\frac{\partial^2 H}{\partial t^2} \end{aligned}\right\} \text{—} (1)$$

The general equ. soln above wave equation

$$\left.\begin{aligned} E(r,t) &= E_0 e^{i(kr-\omega t)} \\ \& \quad B(r,t) &= B_0 e^{i(kr-\omega t)} \end{aligned}\right\} \text{—} (2)$$

Now ∇k

$$\vec{\nabla} \vec{E} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot E_0 e^{i(\bar{k}.\bar{r}-\omega t)}$$

$$r = ix + jy + kz$$

$$\bar{k} = ik_x + jk_y + kk_z$$

~~$\vec{E}_0 = i$~~

$k = i k_x + j k_y + k_z$

$E_0 = i E_{0x} + j E_{0y} + k E_{0z}$

$k \cdot r = k_x \cdot x + k_y \cdot y + k_z \cdot z$

$k \cdot E_0 = k_x \cdot E_{0x} + k_y E_{0y} + k_z \cdot E_{0z}$

further.

$$\vec{\nabla}\cdot\vec{E} = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\,\partial/\partial z\right)\left(i E_{0x} + j E_{0y} + k E_{0z}\right) e^{i(k_x \cdot x + k_y \cdot y + k_z \cdot z - \omega t)}$$

$$= E_{0x}\frac{\partial}{\partial x} e^{i(k_x x + k_y \cdot y + k_z \cdot z - \omega t)} + E_{0y}\frac{\partial}{\partial y} e^{i(\quad)} + E_{0z}\frac{\partial}{\partial z} e^{i(\quad)}$$

$$= i k_x \cdot E_{0x} e^{i(\quad)} + i k_y E_{0y} e^{i(\quad)} + i k_z E_{0z} e^{i(\quad)}$$

$$\nabla\cdot\vec{E} = i[k_x E_{0x} + k_y E_{0y} + k_z E_{0z}) e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$= i k E_0 e^{i(kr - \omega t)}$$

$$\nabla\cdot E = i\vec{k}\cdot\vec{E} \quad \text{—} (1)$$

maxwell first eq^n in free space

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \text{—} (2)$$

$$0 = i\vec{k}\cdot\vec{E}$$

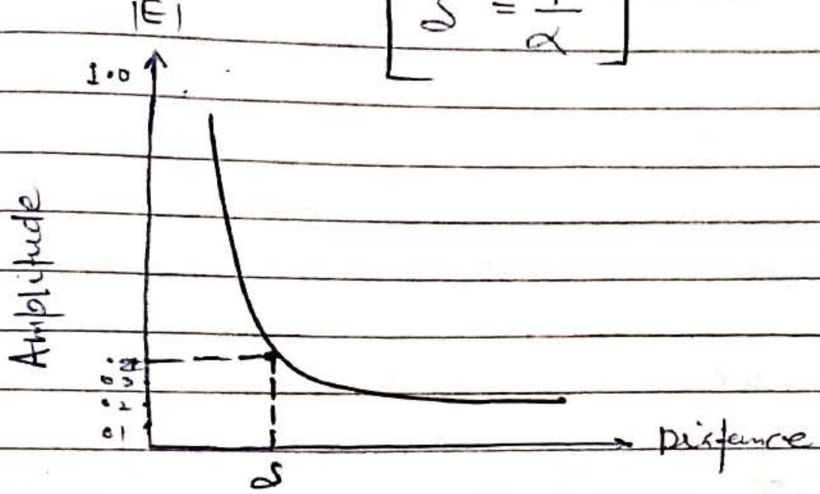
$$[\vec{k}\vec{E} = 0]$$

similarly

$$[\vec{k}\cdot\vec{H} = 0]$$ proved

skin depth (Depth of pentration): Skin depth is define as the depth that which the amplitude of the wave have been reduce by $\frac{1}{e}$ times.

$$\left[\delta = \frac{1}{\alpha} \right]$$



<u>formula.</u> for a good conductor $\frac{\sigma}{\omega\varepsilon} >> 1$

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

* for a poor conductor $\frac{\sigma}{\omega\varepsilon} << 1$

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\delta = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$$

Case Show that the skin depth dependent of frequency for good conductor.

Proof.

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\delta = \frac{1}{\alpha}$$

$$= \frac{1}{\sqrt{\frac{\omega\mu\sigma}{2}}}$$

$$= \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$= \sqrt{\frac{\cancel{2}}{\cancel{2}\pi f\mu\sigma}}$$

$$= \sqrt{\frac{1}{\pi f\mu\sigma}}$$

wave impedance (η): The ratio of the transfer componanet of electric and magnetic field is called wave impedance.

$$\left[\eta = \frac{E}{H}\right]$$

$E_0 = E\sqrt{2}$
& $H_0 = H\sqrt{2}$

or
$$\left[\eta = \frac{E}{H} = \frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.72\ \text{ohm}\right]$$

Radiation pressure: The radiation pressure exerted by the radiations immeted by the sun on a completely absorbing surface.

formula $R.I = \sqrt{\epsilon_r}$

$$V = \frac{c}{R.I}$$

then the maximum energy directly to the surface

$$\left[P = \frac{U}{c} \right]$$

U - Total Energy
c - speed of light

Q1. The permeability, permittivity and conductivity of aluminium are $\mu_r = 1$ $\epsilon_r = 1$ and $\sigma = 3.54 \times 10^7 \, S/m$ find the skin depth ~~if~~ the wave Aluminium.

$f = 71.56 \, MHz$

$f = 71.56 \times 10^6 \, Hz$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$= \frac{1}{\sqrt{3.14 \times 71.56 \times 10^6 \times 4\pi \times 10^{-7} \times 3.54 \times 10^7}}$$

$$\left[\delta = 10 \mu m \right]$$

Q. for a teflon $\mu = \mu_0 = 4\pi \times 10^{-7} \, H/m$ $\epsilon = 70 \epsilon_0$ and conductivity $\sigma = 5 \, S/m$ find the skin depth and action constant of teflon.

$$\delta = \frac{2}{\sigma} \sqrt{\frac{\epsilon}{\mu}}$$

$$= \frac{2}{5} \sqrt{\frac{70 \times 8.85 \times 10^{-12}}{4\pi \times 10^{-7}}}$$

$$\delta = 0.0089 \, m$$

$$\alpha = \frac{1}{\delta} = 112.36 \, Np/m \quad \text{Ans}$$

For a copper $\sigma = 58 mS/m$ for Teflon $\sigma = 30 nS/m$ and $\varepsilon = 2.1\varepsilon_0$ verify that at $f = 1MHz$ copper is good conductor and teflon is good dielectric.

So for good conductor $\frac{\sigma}{\omega\varepsilon} >> 1$

$$\frac{58 \times 10^6}{2\pi f\, 2.1 \varepsilon_0} >> 1$$

$$\frac{58 \times 10^6}{10^6 \times 2 \times 3.14 \times 2.1 \times 8.85 \times 10^{-12}} >> 1$$

$$4.96 \times 10^{12} >> 1$$

for Teflon good dielectric

$$\frac{\sigma}{\omega\varepsilon} << 1$$

$$\frac{30 \times 10^{-9}}{2 \times 3.14 \times}$$

Q Most soil have the conductivity of 10^{-3} S/m & $\varepsilon_r = 2.5$ If electricity field intensity $E = 4.5 \times 10^{-6} \sin(8 \times 10^9 t)$ V/m. Then find the condi current density & displacement current density.

$$\sigma = 10^{-3}\ S/m \qquad \varepsilon_r = 2.5$$

$$E = 4.5 \times 10^{-6} \sin(8 \times 10^9 t)\ V/m$$

$J_c = \sigma E$

$= 10^{-3} \times 4.5 \times 10^{-6} \sin(8\times10^9 t)$

$\left[J_c = 4.5\times10^{-9} \sin(8\times10^9 t) \right] A/m^2$

$J_d = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = \varepsilon_0 \varepsilon_r \frac{\partial E}{\partial t}$

$= \varepsilon_0 \varepsilon_r \frac{\partial}{\partial t} \left(4.5\times10^{-6} \sin(8\times10^9 t) \right)$

$= 8.85\times10^{-12} \times 2.5 \times 4.5\times10^{-6} \times 8\times10^9 \cos(8\times10^9 t)$

$J_d = 796.\times10^{-19} \cos(8\times10^9 t)\ A/m^2$

Q. Determine the conduction current and displacement current having conductivity of 10^{-4} S/m & relative permitivity (ε_r) 2.25 The electric field in the materid is.

$E = 5\times10^{-6} \sin(9\times10^9 t)$ V/m

Solve $\cancel{\sigma = 10^{-4}/\ldots}$ $\sigma = 10^{-4}$ S/m . $\varepsilon_r = 2.25$

$E = 5\times10^{-6} \sin(9\times10^9 t)$

$J_c = \sigma E$

$= 10^{-4} \times 5\times10^{-6} \sin(9\times10^9 t)$

$J_c = 5\times10^{-10} \sin(9\times10^9 t)$]

$J_d = \frac{\partial D}{\partial t}$

In a material for which $\sigma = 5$ S/m & $\epsilon_r = 1$ the electric field intensity $E = 250 \sin(10^{10} t)$ V/m find the conduction & displacement current density & the frequency at which they have equal magnetic.

Solve $\sigma = 5$ S/m $\quad \epsilon_r = 1 \quad E = 250 \sin(10^{10} t)$

$$J = \sigma E$$

$$= 5 \times 250 \sin(10^{10} t)$$

$$= 1250 \sin(10^{10} t)$$

$$J_d = \frac{\partial D}{\partial t} = \frac{\epsilon \partial E}{\partial t} = \epsilon_0 \epsilon_r \frac{\partial E}{\partial t}$$

$$= 8.85 \times 10^{-12} \times 1 \; 250 \cos(10^{10} t) \times 10^{10}$$

$$= 2212.5 \times 10^{-2} \cos(10^{10} t) \text{ A/m}$$

$$\frac{J_C}{J_d} = \frac{\sigma}{\omega \epsilon}$$

if $J_C = J_D$ then

$$\omega \epsilon = \sigma$$

$$2\pi f \epsilon_0 \epsilon_r = \sigma$$

$$f = \frac{\sigma}{2\pi \epsilon_0 \epsilon_r}$$

$$= \frac{5}{2 \times 3.14 \times 8.85 \times 10^{-12} \times 1}$$

$$\left[f = 8.99 \times 10^{10} \text{ Hz} \right]$$

Q. if the magnitude of ~~vector~~ $\vec{H}$ in a plane wave is 1 A/m find the magnitude of E for plane wave in free space.

~~$H_0 = H\sqrt{2}$~~ $\eta = \frac{E_0}{H_0} = 376.72$

$E_0 = 376.72$ V/m

Q. The relative permittivity of distilled water is 81 calculate R.I and velocity of the light wave in it.

$\varepsilon_r = 81$

$R.I = \sqrt{\varepsilon_r}$

$R.I = \sqrt{81}$

$\boxed{R.I = 9}$

* poynting theorem (work energy theorem): At a any point in electromagnetic field the cross product of electric field intensity and magnetic field intensity is a measure of rate of flow of energy at that point.

$$\boxed{\vec{S} = \vec{E} \times \vec{H}}$$

poynting vector :- The rate of flow of energy per unit time per unit area is called pointing vector.

$$\boxed{S = \vec{E} \times \vec{H} = \frac{1}{\mu}(\vec{E} \times \vec{B}) \text{ watt/m}^2}$$

proof. By maxwell's fruther eqn

$$\vec{\nabla} \times \vec{B} = \mu \left[J + \epsilon \frac{\partial E}{\partial t} \right]$$

$$\frac{1}{\mu} (\vec{\nabla} \times \vec{B}) = J + \epsilon \frac{\partial E}{\partial t}$$

$$J = \frac{1}{\mu} (\vec{\nabla} \times \vec{B}) - \epsilon \frac{\partial \vec{E}}{\partial t}$$

Taking dot product of E on both side.

$$\vec{E} \cdot \vec{J} = \frac{1}{\mu} \vec{E} \cdot (\vec{\nabla} \times \vec{B}) - \epsilon \vec{E} \frac{\partial \vec{E}}{\partial t} \quad \text{—— ①}$$

Vector identity

$$\nabla \cdot (\vec{E} \times \vec{B}) = \vec{B} (\vec{\nabla} \times \vec{E}) - \vec{E} (\vec{\nabla} \times \vec{B})$$

$$\vec{E} \cdot (\vec{\nabla} \times \vec{B}) = \vec{B} (\vec{\nabla} \times \vec{E}) - \vec{\nabla} (\vec{E} \times \vec{B}) \quad \text{—— ②}$$

from eqn ① & ② we get

$$\vec{E} J = \frac{1}{\mu} [B \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} (\vec{E} \times \vec{B}) - \epsilon \vec{E} \frac{\partial E}{\partial t}$$

$$\vec{E} \cdot J = -\frac{1}{\mu} \vec{B} \frac{\partial \vec{B}}{\partial t} - \frac{1}{\mu} \vec{\nabla} (\vec{E} \times \vec{B}) - \epsilon \vec{E} \frac{\partial \vec{E}}{\partial t}$$

$$\frac{1}{\mu} \vec{\nabla} (\vec{E} \times \vec{B}) = -\vec{E} \cdot J - \left[\epsilon \vec{E} \frac{\partial \vec{E}}{\partial t} + \frac{1}{\mu} \vec{B} \frac{\partial \vec{B}}{\partial t} \right] \quad \text{—— ③}$$

taking volume integral on both slide we get

$$\frac{1}{\mu} \int_v \nabla (\vec{E} \times \vec{B}) dv = - \int_v \vec{E} \cdot J dv - \int_v \left(\epsilon \vec{E} \frac{\partial \vec{E}}{\partial t} + \frac{1}{\mu} B \frac{\partial \vec{B}}{\partial t} \right) dv \quad \text{—— ④}$$

using Gauss divergence theorem

$$\oint (\vec{E} \times \vec{B}) ds = \int_v \vec{\nabla} (\vec{E} \times \vec{B}) dv \quad \text{—— ⑤}$$

from eqn (4) & (5) we get

$$\left[\frac{1}{\mu} \oint (\vec{E} \times \vec{B}) \, ds = -\int (E \cdot j) \, dv - \int \left(\epsilon \bar{E} \frac{\partial E}{\partial t} + \frac{1}{\mu} \bar{B} \frac{\partial B}{\partial t} \right) dv \right]$$

Q. signification of poyinting vector.

Ans. The poyinting vector help in understanding conservation of energy in electromagnetic wave.

Q. calculate the magnitude of poyinting vector at the surface of the sun given the power radiated by son 5.4×10^{28} W and the radious of the sun is 7×10^8 m

$$P = 5.4 \times 10^{28} \, W$$

$$r = 7 \times 10^8 \, m$$

$$S = \frac{P}{A} = \frac{P}{4\pi r^2}$$

$$\bar{S} = \frac{5.4 \times 10^{28}}{4 \times 3.14 \times (7 \times 10^8)^2}$$

$$S = 875 \times 10^9 \, W/m$$

Q if the avarge distance b/w the ear & sun is 1.5×10^{11} m & and power radiated by sun 5.4×10^{28} W find the avg solor energy incident on the earth.

$$r = 1.5 \times 10^{11} \, m$$

$$P = 5.4 \times 10^{28}$$

$$S = \frac{P}{A} = \frac{5.4 \times 10^{28}}{4\pi \times (1.5 \times 10^{11})^2}$$

$[S = 1.91 \times 10^5 \, W/m^2$

Q. Assuming that all energy from a 1000W lamp is radiated uniformly calculate the average value of intensities electric and magnetic field of radiation at distance of 2m from the lamp.

$$P = 1000\,W \qquad r = 2m$$

$$S = \frac{P}{A}$$

$$S = \frac{\cancel{1000}^{250}}{\cancel{4}\pi (4)}$$

$$= \frac{250}{4\pi}$$

$$= 19.90 \, W/m^2$$

$$S = \frac{P}{4\pi r^2}$$

$$E \cdot H = \frac{1000}{4\pi (2)^2} = \frac{1000}{16\pi} \quad \text{—} ①$$

we know that

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.72 \text{ ohm} \quad \text{—} ②$$

multiply eq ① & ② we get

$$E \cdot H \times \frac{E}{H} = \frac{1000}{16\pi} \times 376.72$$

$$E^2 = 7496.728$$

$$[E = 86.58 \, V/m]$$

$$E_0 = E\sqrt{2}$$
$$= 86.59\sqrt{2}$$
$$\left[E_0 = 122.44 \text{ V/m}\right]$$

putting the value of E in eqn ①

$$H = \frac{1000}{16\pi E}$$

$$H = \frac{1000}{16 \times 3.14 \times 86.59}$$

$$\left[H = 0.23 \text{ A/m}\right]$$

$$H_0 = \sqrt{2} H$$
$$\left[H_0 = 0.32 \text{ A/m}\right]$$

Q. A lamp radiate 500w. power uniformly in all direction. calculate the electric & magnetic field strength at a distance of 5m from lamp.

$$P = 500 \text{ W} \qquad r = 5\text{m}$$

$$S = \frac{P}{A} = \frac{P}{4\pi r^2}$$

$$E \cdot H = \frac{500}{4 \times \pi \times 5 \times 5}$$

$$E H = \frac{500}{100\pi} \text{ —— ①}$$

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.72 \text{ ohm} \text{ —— ②}$$

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multiply eq ① & ② we get

$$E \cdot \cancel{H} \times \frac{E}{\cancel{H}} = \frac{500}{100\pi} \times 376.72$$

$E^2 = 599.87$

$E = 24.49\ V/m$

Q if the earth ·2 cal min-1 cm-2 solar energy. what are the amplitude electric and magnetic field of radiation.

$$|\bar{S}| = |E \times \bar{H}| = E \cdot H \sin 90 = E \cdot H$$

$$|S| = 2\ cal\ min^{-1}\ cm^{-2}$$

$$= \frac{2 \times 4.2}{60 \times (10^{-2})^2}$$

$$= \frac{2 \times 4.2 \times 10^4}{60}$$

$$= \frac{84 \times 10^3}{60}$$

$$= \frac{8400}{6}$$

$$E \cdot H = 1400\ J\ s^{-1}\ m^{-2} \quad \text{—— (I)}$$

we know that

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.72\ ohm \quad \text{—— (II)}$$

multiply eq (1) & (2)

$$E \cdot \cancel{H} \times \frac{E}{\cancel{H}} = 1400 \times 376.72$$

$$E^2 = 527408$$

$E = 726.22\ V/m$

$E_0 = E\sqrt{2}$

$[E_0 = 827.90]$

The sun light skyes the upper atomspenic layer of the earth which energy flux $1.38\ kW/m^2$ what will be the peak value of electric and magnetic feild at that point.

$$|S| = E \cdot H = 130 kW/m^2 = 1380\ W/m^2$$

$$E \cdot H = 1380 \quad \text{---} ①$$

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.72 \quad \text{---} ②$$

multiply ① & ② we get

$E^2 = 1380 \times 376.72$

$E^2 = 5.18 \times 10^6$

$E = 721.02\ W/m$

$E_0 = 1019.67$

$H_0 = \sqrt{2} H$

$H_0 = 1.41 \times 1.913$

$= 2.180\ A/m$

from eq ① we get

$$H = \frac{1380}{721.02}$$

$[H = 1.913\ ]\ A/m$

$E = 726.22 \, V/m$

$E_0 = E\sqrt{2}$

$[E_0 = 827.90]$

The sun light strikes the upper atomspheric layers of the earth which energy flux $1.38 \, kW/m^2$ what will be the peak value of electric and magnetic feild at that point.

$|S| = E \cdot H = 1.38 \, kW/m^2 = 1380 \, W/m^2$

$E \cdot H = 1380$ — ①

$\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.72$ — ②

multiply ① & ② we get

$E^2 = 1380 \times 376.72$

$E^2 = 5.18 \times 10^6$

$E = 721.02 \, w/m$

$E_0 = 1019.67$

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$H_0 = 1.41 \times 1.913$

$= 2.180 \, A/m$

from eq ① we get

$H = \frac{1380}{721.02}$

$[H = 1.913] \, A/m$

Shorttypequestion:

physicall significance:

① $\nabla \cdot D = \rho$ It is signifies that the Gauss law in electrostatics for the static charge.

② $\nabla \cdot B = 0$ magnetic monopole doesnot exist in nature.

③ $\nabla \times E = -\frac{\partial B}{\partial t}$. An electric field is produced by a Changing magnetic field.

④ $\nabla \times B = \mu \left[J + \frac{\partial D}{\partial t} \right]$ A conduction current as well as changing electric flux. produces a magnetic field.

Q. show that magnetic nature pole does not exist the nature.

Ans.

Q. what is quantom ~~quantiymum~~ confinment in nano material.

Ans. Quantom ~~quantiyies~~ Confinment: phenomena that occur so that when the electronic properties of material are alteratte by reducing its size.

Q. Defference between step index and Graded index fibre.

① The refrective index of pole is uniform but suddenly change at core cladding interface.	① The refrective index of core is gradually decreases
② The path of light propagete is zig-zag manner.	② The path of light is propogate in helical.

	Distoration
(1) Discurrion is more in multimode fibre	Discauses is less.
(4) Nomerical apxature is more multimode fiber.	(4) Nomerical apxature is less
(4) ~~using maxwell equation~~ curl ~~of F~~	

Q. using ~~curl e~~ maxwell equation ~~curl~~.

$$\bar{\nabla} \times \bar{B} = u\left[J + \frac{\partial D}{\partial t}\right] \quad \text{prove that} \quad \bar{\nabla}\cdot\bar{D} = \rho$$

Solve Given that

$$\nabla \times B = u_0\left[J + \frac{\partial D}{\partial t}\right]$$

taking Div on both side we get.

$$\nabla\cdot(\nabla\times B) = u_0\left[\nabla\cdot J + \frac{\partial}{\partial t}(\nabla\cdot D)\right]$$

$\because \nabla\cdot(\nabla\times B) = 0$ vector identity

$$\nabla\cdot J + \frac{\partial}{\partial t}(\nabla\cdot D) = 0 \quad \text{—} (1)$$

From eqn of continuity

$$\nabla\cdot J + \frac{\partial \rho}{\partial t} = 0$$

$$\nabla\cdot J = -\frac{\partial \rho}{\partial t} \quad \text{—} (2)$$

from eqn (1) & (2) we get

$$-\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial t}(\nabla\cdot D) = 0$$

$\therefore \frac{\partial}{\partial t}(\nabla \cdot D) = \frac{\partial \rho}{\partial t}$

Comparing $[\nabla \cdot D = \rho]$

Q. using maxwell eqn $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$ Derive columbms. law of electrostatics.

Solve: Given that $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$ —①

taking volume integral

$$\int \nabla E \, dv = \frac{1}{\varepsilon_0} \int \rho \, dv \quad —②$$

using Gauss divergence theorem.

$$\oint E ds = \int (\nabla \cdot E) dv \quad —③$$

from ② & ③ we get

$$\oint E ds = \frac{1}{\varepsilon_0} \int \rho dv$$

$$\oint E ds = q/\varepsilon_0 \quad —④$$

for spherical surface of radius 'r' around the charge 'q' is

$$\oint E ds = E(4\pi r^2) \quad —⑤$$

from ④ & ⑤ we get

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi r^2} \frac{q}{\varepsilon_0} \quad —⑥$$

the force on charge 'q_0' in the electric field of charge is $F = q_0 E \rightarrow ⑦$

from ⑥ & ⑦:

$$\left[F = \frac{1}{4\pi\varepsilon_0} \frac{q q_0}{r^2} \right]$$

This is known as Coulomb's law.

$|S| = 2 \, cal \, min^{-1} cm^{-2}$

$$= \frac{2 \times 4.12}{60 \times (10^{-2})^2}$$

$E \cdot H = 1400 \, J \, s^{-1} m^{-2}$ — (I)

we know that

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.76 \text{ — (II)}$$

$E^2 = 1400 \times 376.76$

$E = 726.22 \, V/m$

$\cancel{E = E_0}$ $\quad E_0 = E\sqrt{2}$

$E_0 = 1027.23 \, V/m$